

## GGSIPIU mathematics 2011

1. Let  $f(x) = ax + b$ ,  $a < 0$ , then  $f^{-1}x = fx$ , "  $x$  if and only if

- a  $a = -1, b \in \mathbb{R}$       b  $a = -1, b = 4$   
 c  $a = -3, b \in \mathbb{R}$       d None of these

2. The domain of  $\cos^{-1} \frac{x-3}{2} - \log_{10} 4 - x$  is

- a  $1, 4$       b  $[1, 4]$   
 c  $1, 4]$       d  $[1, 4]$

3. If  $f(x)$  is a polynomial function of the second degree such that  $f(-3) = 6$ ,  $f(0) = 6$  and  $f(2) = 11$ , then the graph of the function  $f(x)$  cuts the ordinate  $x = 1$  at the point

- a  $1, 8$       b  $1, 4$   
 c  $1, -2$       d None of these

4. Let  $A$  and  $B$  be two sets, then  $A \cap B' \cap A' \cap B$  is equal to

- a  $A'$       b  $A$   
 c  $B'$       d None of these

5. The mean of 10 observations is 16.3. By an error one observation is registered as 32 instead of 23. Then, the correct mean is

- a 15.6      b 15.4  
 c 15.7      d 15.8

6. Mean deviation of 6, 8, 12, 15, 10, 9 through mean is

- a 10      b 2.33  
 c 2      d None of these

7. The image of the point  $(2, 1)$  w.r.t. the line  $x + 1 = 0$  is

- a  $(2, 5)$       b  $(0, 5)$   
 c  $(-4, 1)$       d  $(-2, -3)$

8. The value of  $x$  which satisfies  $81 + \cos x + \cos 2x + \dots = 64$  in  $[-\pi, \pi]$  is

$$\begin{array}{ll} \text{a} & \frac{p}{2}, \frac{p}{3} \\ \text{b} & \pm \frac{p}{2}, \pm \frac{p}{6} \\ \text{c} & \pm \frac{p}{2}, \pm \frac{p}{3} \\ \text{d} & \pm \frac{p}{6}, \pm \frac{p}{3} \end{array}$$

9. If  $d = l a \times b + m b \times c + n c \times a$  is equal to  $\frac{1}{8} [a \times b + b \times c + c \times a]$ , then  $l+m+n$

- a  $d \cdot a+b+c$       b  $2d \cdot a+b+c$   
c  $4d \cdot a+b+c$       d  $8d \cdot a+b+c$

10. The area of the triangle formed by the points whose position vectors are  $3i + j$ ,  $5i + 2j + k$ ,  $i - 2j + 3k$  is

- a  $\sqrt{23}$  sq units  
b  $\sqrt{21}$  sq units  
c  $\sqrt{29}$  sq units  
d  $\sqrt{33}$  sq units

11. If 1, -2, -2 and 0, 2, 1 are direction ratios of two lines, then the direction cosines of a line perpendicular to both the lines are

- a  $\left(\frac{1}{3}, -\frac{1}{3}, \frac{2}{3}\right)$   
b  $\left(\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}\right)$   
c  $\left(-\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$   
d  $\left(\frac{-2}{\sqrt{14}}, \frac{1}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$

12. The length of the normal to the curve  $y = a \cosh \left(\frac{x}{a}\right)$  at any point varies as

- a ordinate  
b abscissa  
c square of the abscissa  
d square of the ordinate

13. The slope of the tangent to the curve  $y = \frac{x}{1+x^3}$  at the point where  $x=1$  is

- a  $\frac{1}{4}$       b  $\frac{1}{2}$       c  $\frac{3}{4}$       d  $\frac{5}{4}$

c 1 d None of these

14. If  $f(x) = a \log_e |x| + bx^2 + x$  has extremum at  $x = 1$  and  $x = 3$ , then

a  $a = -3/4, b = -1/8$

b  $a = 3/4, b = -1/8$

c  $a = -3/4, b = 1/8$

d None of the above

15. In the expansion of  $\left(x^3 + \frac{1}{x^2}\right)^n$ , if the sum of the coefficient of  $x^5$  and  $x^{10}$  is 0, then  $n$  is

a 25 b

c 15 d

16. Let  $z_1, z_2$  be two roots of the equation  $z^2 + az + b = 0$ ,  $z$  being complex number. Further assume that the origin,  $z_1$  and  $z_2$  form an equilateral triangle. Then,

a  $a^2 = b$  b  $a^2 = 2b$

c  $a^2 = 3b$  d  $a^2 = 4b$

17. A square is inscribed in the circle  $x^2 + y^2 - 2x + 4y - 3 = 0$  with its sides parallel to the coordinate axes. One vertex of square is

a 3, 4 b 3, -4

c 8, -5 d -8, 5

18. If  $f: \mathbb{R} \rightarrow \mathbb{R}$  is continuous such that  $f(x+y) = f(x) + f(y)$ ,  $\forall x, y \in \mathbb{R}$  and  $f(1) = 2$ , then  $f(100)$  equals to

a 100 b 50

c 200 d 0

19.  $f(x) = x \sin \frac{1}{x}$  is

a continuous but not differentiable at  $x = 0$

b discontinuous but differentiable at  $x = 0$

c differentiable at  $x = 0$

d can not be determined

20.  $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 - bc & b^2 - ca & c^2 - ab \end{vmatrix}$  equals to

a 0                      -                      -

b 1                      -c c

c abc

d a    -b b                      -a

21. The sum  $\cos 10^\circ + \cos 20^\circ + \cos 30^\circ + \dots + \cos 1790^\circ + \cos 1800^\circ$  is equal to

a 0                      b 1

c -1                      d 2

22. If  $a, b, c$  are in GP and  $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$ , then  $x, y, z$  are in

a AP                      b GP

c HP                      d None of these

23. If  $A$  is a square matrix such that  $A^2 = I$ , then  $A^{-1}$  is equal to

a I                      b O

c A                      d  $I + A$

24. 5th term from the end in the expansion of  $\left(\frac{x^2}{2} - \frac{2}{x^2}\right)^{12}$  is

a  $-7920x^{-4}$     b  $7920x^{-4}$                       -

c  $7920x^{-4}$                       d  $-7920x^4$

25. Which of the following is not a logical statement?

a 8 is less than 6

b every set is a finite set

c Kashmir is far from here

d the sun is a star

26.  $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3$  is equal to

a 0                      b  $\pi$

c  $\frac{p}{2}$  d None of these

27.  $\int_0^{\infty} \frac{1}{1+e^x} dx$  is equal to

a 0 b  $\frac{p}{2}$

c  $\log 2 - 1$  d  $-\log 2$

28. If  $|a| = 8$ ,  $|b| = 3$  and  $|a+b| = 12$ , then the value of a.b is

a 6 or -6 b  $12\sqrt{3}$  or  $-12\sqrt{3}$

c 8 or -8 d None of these

29. The value of  $\sum_{n=0}^{\infty} C_n$  is

a 1 b 0

c  $2^n$  d n

30. Coefficient of variation of two distributions are 50% and 60% and their arithmetic means are 30 and 25 respectively. Difference of their standard deviation is

a 1 b 1.5

c 2.5 d 0

31. If  $i, j, k$  are the usual three perpendicular unit vectors then the value of  $i \cdot (j \times k) + j \cdot (k \times i) + k \cdot (i \times j)$  is

a 0 b -1

c 3 d 1

32. The solution of  $y dx - x dy + \log x dx = 0$  is

a  $y - \log x - 1 = Cx$

b  $x + \log y + 1 = Cx$

c  $y + \log x + 1 = Cx$

d  $y + \log x - 1 = Cx$

33. Which of the following differential equation has  $y = c_1 e^x + c_2 e^{-x}$  as the general solution ?

a  $\frac{d^2 y}{dx^2} + y = 0$  b  $\frac{d^2 y}{dx^2} - y = 0$

c  $\frac{d^2y}{dx^2} + 1 = 0$       d  $\frac{d^2y}{dx^2} - 1 = 0$

34.  $\int \frac{1}{\sin(x-a)\sin(x-b)} dx$  is equal to

- a  $\frac{1}{\sin(b-a)} \log \left| \frac{\sin(x+b)}{\sin(x+a)} \right| + C$   
 b  $\frac{1}{\sin(b+a)} \log \left| \frac{\sin(x-b)}{\sin(x-a)} \right| - C$   
 c  $\frac{1}{\sin(b+a)} \log \left| \frac{\sin(x-b)}{\sin(x-a)} \right| + C$   
 d None of the above

35.  $\int \frac{dx}{x^2\sqrt{4-x^2}}$  is equal to

- a  $\frac{1}{4} \left( \frac{\sqrt{4-x^2}}{x} \right) + C$       b  $\frac{1}{2} \left( \frac{\sqrt{4-x^2}}{x} \right) + C$   
 c  $-\frac{1}{4} \left( \frac{\sqrt{4-x^2}}{x} \right) + C$       d  $-\frac{1}{2} \left( \frac{\sqrt{4-x^2}}{x} \right) + C$

36. If  $\tan^{-1}2$ ,  $\tan^{-1}3$  are two angles of a triangle, then the third angle is

- a  $30^\circ$       b  $45^\circ$   
 c  $60^\circ$       d  $75^\circ$

37.  $\lim_{x \rightarrow 0} \left( \frac{16^x + 9^x}{2} \right)^{1/x}$  is equal to

- a  $25/2$       b  $12$       c  $12$       d  $12$

38. Let  $a = \min\{x^2 + 2x + 3, x^2 + 1\}$  and  $b = \lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta^2}$ . The value of  $\sum_{r=0}^n a^r \cdot b^{n-r}$  is

- a  $\frac{2^{n+1} - 1}{3 \cdot 2^n}$       b  $\frac{2^{n+1} + 1}{3 \cdot 2^n}$   
 c  $\frac{4^{n+1} - 1}{12^n}$       d None of these

39. The matrix  $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$  is a

- a diagonal matrix  
 b symmetric matrix  
 c skew-symmetric matrix

d identity matrix

40. A teacher takes 3 children from her class to the zoo at a time as often as she can, but she does not take the same three children to the zoo more than once. She finds that she goes to the zoo 84 times more than a particular child goes to the zoo. The number of children in her class is

- a 12      b 10  
c 60      d None of these

41. If A = -3, 4, B = -1, -2, C = 5, 6, D = x, -4 are vertices of a quadrilateral such that  $\angle DAB = 2\angle DAC$ . Then, x is equal to

- a 6      b 9      c 69      d 96

42. The area of the parallelogram formed by the points (1, 1, 1), (-1, 5, 5), (2, 2, 5) is

- a 81      b 9  
c 36      d 18

43. If  $f(x) = \frac{9^x}{9^x + 3}$ , then  $f\left(\frac{1}{2012}\right) + f\left(\frac{2}{2012}\right) + \dots + f\left(\frac{2011}{2012}\right)$  is equal to

- a 1005      b 1005.5  
c 1006      d 1006.5

44.  $\frac{1}{\sin^2 101^\circ} \cdot \sec 101^\circ$

- a 0      b 2  
c -1      d 1

45.  $\tan^{-1}\left(\frac{1}{1+2}\right) + \tan^{-1}\left(\frac{1}{1+(2)(3)}\right) + \dots + \tan^{-1}\left(\frac{1}{1+n(n+1)}\right) \equiv \tan^{-1}q$

- a  $\frac{n}{n+1}$       b  $\frac{n+1}{n+2}$   
c  $\frac{n+2}{n+1}$       d  $\frac{n}{n+2}$

46. If A is  $3 \times 3$  and  $\det A = 6$ , then  $\det 2 \operatorname{adj} A$  is equal to

- a 48      b 8  
c 288      d 12

47. The probability that a leap year has only 52 Sundays is

- a  $\frac{4}{7}$       b  $\frac{5}{7}$   
c  $\frac{6}{7}$       d  $\frac{1}{7}$

48. If  $\int \frac{2^x}{1-4^x} dx = \log \sin^{-1} 2x + C$ , then  $\log 2$  equals to

- a  $\log 2$       b  $\frac{1}{2} \log 2$   
c  $\frac{1}{2}$       d  $\frac{1}{\log 2}$

49. If S is circumcenter, G the centroid, O the orthocenter of  $\triangle ABC$ , then  $SA+SB+SC$  is equal to

- a SG      b OS  
c SO      d OG

50. The centre and radius of the sphere  $(x^2+y^2+z^2-2x+4y-5k)+1=0$  are

- a  $(3i+4j-5k, 1)$       b  $(-3i-4j+5k, 7)$   
c  $(-3i-4j+5k, 7)$       d  $(3i+4j-5k, 7)$