KEAM 2023 Solved Paper

Physics & Chemistry

Question 1

A projectile is thrown at a speed V and at an angle θ with the horizontal. If the speed at its maximum height is $\frac{V}{\sqrt{3}}$, then the value of tan θ is :

Options:

A. $\sqrt{3}$

B. 1

$\frac{}{\sqrt{3}}$

C. $2\sqrt{2}$

D. 3

E. $3\sqrt{3}$

Answer: C

Solution:

Solution:

u cos θ

v cos θ= $\frac{v}{3}$

$\cos\theta = \frac{1}{3}$

$\tan\theta = \sqrt{\sec^2\theta - 1}$

$\sqrt{3^2 - 1} = \sqrt{8} = 2\sqrt{2}$

Question 2

Consider a vector addition $\vec{P} + \vec{Q} = \vec{R}$.

If $\vec{P} = |P\hat{i}| + |Q\hat{j}|$ and $|\vec{R}| = 3|\vec{P}|$, then $|\vec{Q}|$ is :

Options:

A. $-\sqrt{10}$

B. 30

C. $\sqrt{30}$

D. $2\sqrt{10}$

E. $2\sqrt{20}$

Answer: A

Solution:

Solution:

$\vec{P} + \vec{Q} = \vec{R}$

$\vec{P}i + 10\hat{n} = 3p\hat{j}$

$|10\hat{n}| = |3p\hat{j} - pi|$

$10^2 = 9p^2 + p^2$

$100 = 10p^2$

Question 3

A car is moving with an initial speed of 5m / s. A constant braking force is applied and the car is brought to rest in a distance of 10m. What is the average speed of the car during the deceleration process?

Options:

A. 1m / s

B. 2.5m / s

C. 4m / s

D. 5m / s

E. 7m / s

Answer: B

Solution:

Solution:

Question 4

Consider a particle executing a simple harmonic motion. Let x, A, K and U are displacement, amplitude, kinetic energy and potential energy, respectively, of the particle at certain instant of time. If $\frac{K}{U} = 3$, then $\frac{x}{A}$ is :

Options:

A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. $\frac{3}{2}$

D. $\frac{1}{9}$

E. $\frac{4}{9}$

Answer: B

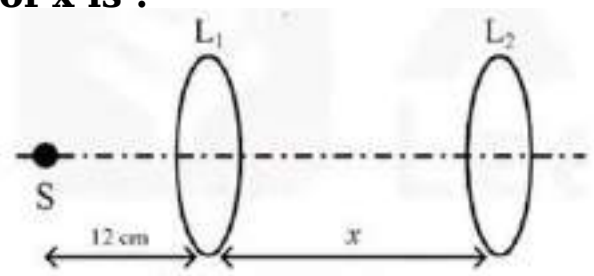
Solution:

Solution:

Question 5

Two thin convex lenses L

1 and L2 have focal lengths 4 cm and 10 cm, respectively. They are separated by a distance of x cm as shown in the figure. A point source S is placed on the principal axis at a distance 12 cm to the left of L1. If the image of S is formed at infinity, the value of x is :



Options:

A. 6 B. 16

C. 14 D. 24

E. 10

Answer: B

Solution:

Solution:

Question 6

What is the de Broglie wavelength corresponding to a ball of mass 100g moving with a speed of 33m / s ? (Plank's constant = 6.6 × 10−34J / s)

Options:

A. 1×10^{-34}m

B. 2×10^{-34}m

C. 3×10^{-34}m

D. $1 \times {}^{34}_{10}$m

E. $2 \times {}^{34}_{10}$m

Answer: B

Solution:

Solution:

Question 7

A laser source emits light of wavelength 300 nm and has a power of 3.3 mW. The average number of photons emitted per second is : (Speed of light $= 3 \times 10^{6}$m s^{-1}, Plank's constant $= 6.6 \times 10^{-34}$J/ s)

Options:

A. 2×10^{15}

B. 1×10^{15}

C. 5×10^{15}

D. 3×10^{15}

E. 4×10^{15}

Answer: C

Solution:

Solution:

Question 8

A thin convex lens of refractive index 1.5 has a focal length of 10 cm in air. When the lens is immersed in a fluid, its focal length becomes 70 cm. The refractive index of the fluid is :

Options:

A. 1.33

B. 1.6

C. 1.25

D. 1.45

E. 1.4

Answer: E

Solution:

Solution:

Question 9

For the hydrogen spectrum, the wavelength in Balmer series is given by $\frac{1}{\lambda} = R\left(\frac{1}{n1^2} - \frac{1}{n2^2}\right)$ where λ= wavelength and R is Rydberg constant. What are the values of n1 and n2 for the longest wavelength in the Balmer series ?

Options:

A. $n_1 = 2,\ n_2 = 3$

B. $n_1 = 2,\ n_2 = 4$

C. $n_1 = 1,\ n_2 = 2$

D. $n_1 = 2,\ n_2 = \infty$

E. $n_1 = 3,\ n_2 = \infty$

Answer: A

Solution:

Solution:

Question 10

Car P is heading east with a speed V and car Q is heading north with a speed $\sqrt{3}$V . What is the velocity of car Q with respect to car P ?

Options:

A. V $\sqrt{3}$, heading north

B2V, ∘. 30 east of north

C. V√, ¯∘360 west of north

D. $2V$, 30 ̊west of north

E. $V\sqrt{2}$, 45 ̊west of north

Answer: D

Solution:

Solution:

Question 11

A particle at rest starts from the origin with a constant acceleration $\vec{a}$ that makes an angle 60° with the positive y-axis. If its displacement along y-axis is 10m in time 2 s, then the magnitude of $\vec{a}$ is :

Options:

A. $10ms^{-2}$

B. $4ms^{-2}$

C. $8ms^{-2}$

D. $15ms^{-2}$

E. $20ms^{-2}$

Answer: A

Solution:

Solution:

Question 12

Suppose a force is given by the expression $= kx^2$; where x has the dimension of length. The dimension of k is :

Options:

A. $ML^{1}T^{-1}$

B. MLT^{-1}

C. MT^{-2}

D. $M^{-1}L^{1}T^{-2}$

E. $ML^{-1}T^{1}$

Answer: E

Solution:

Solution:

Question 13

A horizontal force is exerted on a 20 kg box to slide it up on an inclined plane with an angle of 30˚. The frictional force retarding the motion is 80N. If the box moves with a constant speed, then the magnitude of the force is : (Take g = $10ms^{-2}$)

Options:

A. $50\sqrt{2}$ N

B. 100N

C. $80\sqrt{3}$ N

D. $100\sqrt{2}$ N

E. $120\sqrt{3}$ N

Answer: E

Solution:

Solution:

Question 14

In a Young's double slit experiment which of the following statements is NOT true?

Options:

A. Angular separation of the fringes remains constant when the screen is moved away from the plane of the slits.

B. Fringe separation increases when the separation between the two slits decreases.

C. Sharpness of the fringe pattern decreases when the source slit width increases.

D. Distance between the fringes decreases when the separation between slits and the screen increases.

E. The central fringe is white when the monochromatic source is replaced by a white light source.

Answer: D

Solution:

Solution:

Question 15

N capacitors, each with 1μF capacitance, are connected in parallel to store a charge of 1C. The potential across each capacitor is 100V. If these N capacitors are now connected in series, the equivalent capacitance in the circuit will be :

Options:

A. 10^{-4}F

B. 10^{-6}F

C. 10^{-10}F

D. 5×10^{-8}F

E. 10^{-2}F

Answer: C

Solution:

Solution:

Question 16

A train consists of an engine and 3 coaches, first coach is closest to the engine, third coach is farthest from engine. The train is moving with a constant acceleration a. The mass of each coach is M . The force exerted by the first coach on the second coach will be :

Options:

A. M a B. 2M a C. 3 Ma D. 4 Ma

E. √2M a

Answer: B

Solution:

Solution:

Question 17

A uniform thin rod of mass 3 kg has a length of 1m. If a point mass of 1 kg is attached to it at a distance of 40 cm from its center, the center of mass shifts by a distance of:

Options:

A. 2.5 cm B. 5 cm C. 8 cm D. 10 cm E. 20 cm

Answer: D

Solution:

Solution:

Question 18

A wheel is rolling on a plane surface. A point on the rim of the wheel at the same level as the centre has a speed of 4m / s. The speed of the centre of the wheel is :

Options:

A. 4m / s

B. 0

C. 2√2m / s

D. 8m / s

E. 4√2m / s

Answer: C

Solution:

Solution:

Question 19

An unpolarised light is incident on a glass slab such that the reflected ray is totally polarised. If the angle of refraction is 30˚, the refractive index of the glass is :

Options:

A. 1.5 B. 1.73 C. 1.41 D. 1.45 E. 1.60

Answer: B

Solution:

Solution:

Question 20

A planet has an escape speed of 10 km / s.
The radius of the planet is 10, 000 km.
The acceleration due to gravity of the planet at its surface is :

Options:

0/2A. 1m s

. /2B 9.8m s

/2C. 20m s

/2D. 2.5m s

/2E. 5m s

Answer: E

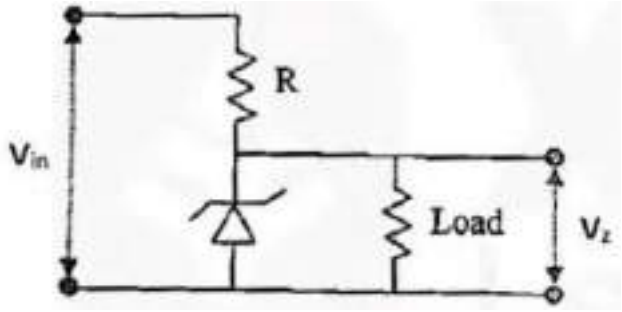
Solution:

Solution:

Question 21

In a Zener regulated power supply circuit as shown in figure below, a Zener diode with V_z=10V is used for regulation. The load current, Zener current and unregulated input V_{in} are 5mA,35mA and 20V,

respectively. The value of R is :



Options:

A. 1000Ω

B. 750Ω

C. 250Ω

D. 100Ω

E. 500Ω

Answer: C

Solution:

Solution:

Question 22

An average frictional force of 80N is required to stop an object at a distance of 25m. If the initial speed of the object is 20m / s, the mass of the object is :

Options:

A. 25 kg B. 12 kg C. 30 kg D. 40 kg E. 10 kg

Answer: E

Solution:

Solution:

Question 23

An ideal gas is kept in a closed container. If the temperature is doubled and the volume of the container is reduced to half, the gas pressure is :

Options:

A. unchanged

B. halved

C. doubled

D. increased by 4 times

E. increased by 16 times

Answer: D

Solution:

Solution:

Question 24

A metal wire of natural length 50 cm and cross-sectional area $4.0mm^2$ is fixed at one end. A mass of 2.4 kg is hung from the other end of the wire. If the elastic potential energy of the wire is $1.8\times\ 10^{-4}$J, then its Young's modulus is : (Take g = $10ms^{-2}$)

Options:

A. $1.6\times 10^{11}Nm^{-2}$

B. $2.4\times 10^{11}Nm^{-2}$

C. $3.2\times 10^{11}Nm^{-2}$

D. $1.8\times 10^{11}Nm^{-2}$

E. $2.0\times 10^{11}Nm^{-2}$

Answer: E

Solution:

Solution:

Question 25

Select the incorrect statement about friction:

Options:

A. Static friction force is always equal to μN, where μ is co-efficient of static friction and N is normal force.

B. Friction is a non-conservative force.

C. Friction arises from electro-magnetic force.

D. Friction always opposes relative motion between two surfaces.

E. Maximum value of static friction is μN, where μ is co-efficient of static friction and N is normal force.

Answer: A

Solution:

Solution:

Question 26

The angle of minimum deviation for a prism of apex angle 60° and refractive index of $\sqrt{2}$ is:

Options:

A. 45°

B. 90°

C. 30°

D. 60°

E. 15°

Answer: C

Solution:

Solution:

Question 27

An ideal diatomic gas is made up of molecules that do not vibrate. Its volume is compressed by a factor of 32 , without any exchange of heat. If the initial and final pressures are P_1 and P_2 respectively, the ratio $P_1:P_2$ is :

Options:

A. 7 : 5

B. 128 : 1

C. 1 : 32

D. 32 : 1

E. 1 : 128

Answer: E

Solution:

Solution:

Question 28

A body is moving in a straight line under the influence of a source of constant power. If its displacement at time t = 0 and 10 s are 0 and 10m, respectively. The displacement at time t = 20 s is :

Options:

A. 20m

B. 40m

C. $10\sqrt{2}$ m

D. $20\sqrt{2}$ m

E. $5\sqrt{10}$ m

Answer: D

Solution:

Solution:

Question 29

A glass capillary of radius 0.15 mm is dipped into a liquid of density and surface tension 1600 kg / m3 and $0.12Nm^{-1}$, respectively. The liquid in the capillary rises by a height of 5.0 cm. The contact angle between liquid and glass will be : (Take g = $10ms^{-2}$)

Options:

A. 30°

B. 0°

C. 45°

D. 75˚

E. 60˚

Answer: E

Solution:

Solution:

Question 30

A gun fires N bullets per minute. The mass of each bullet is 10g and every bullet travels with a speed of 600m / s. If the power delivered by the gun is 9000W, the value of N is :

Options:

A. 300

B. 400

C. 360

D. 420

E. 250

Answer: A

Solution:

Solution:

Question 31

In an oil drop experiment, ' n ' numbers of electrons are stripped from an oil drop to make it positively charged. A vertical electric field of magnitude 4.9×10^{14}N/C is applied to balance the force due to gravity on the oil drop. If the mass of oil drop is 80µg, the value of ' n ' will be: (Take g = 9.8m/s^2 and charge of an electron $= 1.6 \times 10^{-19}$C)

Options:

A. 1

B. 10

C. 100

D. 1000

E. 10000

Answer: B

Solution:

Solution:
Question is wrong - None of above

Question 32

A radioactive nuclei has a half life of 693 s. The activity of one mole of that nuclei sample is : (Avogadro's number $= 6.023 \times 10^{23}$ and ln(2) = 0.693)

Options:

A. 2×10^{10} Bq

B. $3.7\times\ 10^{10}$ Bq

C. $6.023\times\ 10^{20}$

D. $0.5\times\ 10^{-10}$ Bq

E. 1×10^{20} Bq

Answer: C

Solution:

Solution:

Question 33

A projectile is thrown at an angle 60◦ above the horizontal and with kinetic energy 40 J. The kinetic energy of the projectile at the highest point of its trajectory will be :

Options:

A. 10J

B. 40J

C. 20J

D. $20\sqrt{2}$ J

E. $20\sqrt{3}$ J

Answer: A

Solution:

Solution:

Question 34

A billiard ball B1 moving with velocity V, collides with another billiard ball B2 at rest. After the collision, ball B1 is deflected by 60̊ and the angle between the velocities of these two balls is 90˚. The speed of the ball B2 after the collision is :

Options:

A. $\frac{V}{2}$

B. $\frac{3V}{2}$

C. 2V

D. $\frac{2V}{\sqrt{3}}$

E. $\frac{\sqrt{3}V}{2}$

Answer: E

Solution:

Solution:

Question 35

Two satellites A and B are moving around the earth in a circular orbit of radius ' R ' and ' 2R ', respectively. If the kinetic energy of the satellite A is two-times the kinetic energy of the satellite B, the ratio of their masses (mA : mB) is :

Options:

A. 1 : 2 B. 2 : 1 C. 1 : 1 D. 1 : 4 E. 4 : 1

Answer: C

Solution:

Solution:

Question 36

An object at rest suddenly explodes into three parts of equal masses. Two of them move away at right angles to each other with equal speed of 10m / s. The speed of the third part just after the explosion will be :

Options:

A. 10m / s

B. 20m / s

C. $2\sqrt{10}$m / s

D. 0

E. $10\sqrt{2}$m / s

Answer: E

Solution:

Solution:

Question 37

Two identical solid spheres, each of radius 10 cm, are kept in contact. If the moment of inertia of this system about the tangent passing through the point of contact is 0.14 kg · m2, then mass of each sphere is

Options:

A. 5 kg

B. 17.5 kg

C. 35 kg

D. 2.5 kg

E. 10 kg

Answer: A

Solution:

Solution:

Question 38

A NOR gate has two input I 1 and I 2 and one output terminal Y . Which of the following configuration (truth table) is INCORRECT for the NOR gate?

Options:

A. I

B. I

$_1$ =0, I $_2$ = 0, Y = 1

$_1$ =0, I_2 = 0, Y =

C. I_1 =1, I_2 = 1, 0

D. I_1 =1, I_2 = 0 , Y =

E. I $_1$ =0, I $_2$ = 1, 0Y = 0

Answer: B Y =

Solution: 0

Solution:

Question 39

The kinetic energy of a particle of mass m 1 moving with a speed V is same as the kinetic energy of a solid sphere of mass m 2 rolling on the plane surface. If the speed of the centre of the sphere is also V, then $\frac{m_1}{m_2}$ is :

Options:

A. $\frac{7}{10}$

B. $\frac{1}{2}$

$\frac{2}{5}$

C. $\frac{7}{7}$

D. $\frac{5}{2}$

$\frac{2}{3}$

E. –

Answer: D

Solution:

Solution:

Question 40

Line-of-sight communication happens by means of:

Options:

A. Ground wave

B. Sky wave

C. Surface wave

D. Space wave

E. Seismic wave

Answer: D

Solution:

Solution:

Question 41

A ring of radius 1.75m stands vertically. A small sphere of mass 1 kg rolls on the inside of this ring without slipping. If it has a velocity of 10m / s at the bottom of the ring, then its velocity when it reaches the top is : (Take g = 10m s^{-2})

Options:

A. $3\sqrt{2}$m / s

B. $2\sqrt{3}$m / s

C. $8\sqrt{2}$m / s

D. $2\sqrt{5}$m / s

E. $5\sqrt{2}$m / s

Answer: E

Solution:

Solution:

Question 42

A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 5 MHz. The frequencies of the side bands are :

Options:

A. 4.5 MHz and 5.5 MHz

B. 4.95 MHz and 5.05 MHz

C. 4.995 MHz and 5.005 MHz

D. 4.9995 MHz and 5.0005 MHz

E. 5 MHz and 5 kHz

Answer: C

Solution:

Solution:

Question 43

A string clamped at both the ends has a mass 10 gm, length 1m and it is kept under tension of 1N. It is vibrating in the fundamental mode with an amplitude of 1 cm. Assuming the standing wave pattern, the maximum acceleration seen in the string is :

Options:

A. $4\pi^2$ m / s^2

B. $2\pi^2$ m / s^2

C. π^2 m / s

D. $4\pi^2$ m / s^2

E. $2\pi^2$ m / s^2

Answer: C

Solution:

Solution:

Question 44

A spherical ball is subjected to a pressure of 100 atmosphere. If the bulk modulus of the ball is 10^{11} N/m^2, then change in the volume is :

Options:

A. $10^{-1\%}$

B. $10^{-2\%}$

C. $10^{-3}\%$

D. $10^{-4\%}\%$

E. $10^{-5}\%$

Answer: B

Solution:

Solution:

Question 45

A hollow sphere of radius ' r ' encloses an electric dipole composed of two charges + q and − q. The net flux of electric field through the surface of the sphere due to the enclosed dipole is :

Options:

A. $\frac{2q}{\varepsilon_0}$

B. $2q\frac{1}{\varepsilon_0}.4\pi r^2$

C. infinite

D. zero

E. $q\frac{1}{\varepsilon_o}$

Answer: D

Solution:

Solution:

Question 46

The work done W is required by an agent to form a bubble of radius R. An extra amount of work Δ W is required to increase the radius by Δ R. If $\frac{\Delta R}{R}$ = 1%, then $\frac{\Delta W}{W}$ is :

Options:

A. 2%

B. 1%

C. 4%

D. 3%

E. 0.5%

Answer: A

Solution:

Solution:

Question 47

Each side of a regular hexagon has resistance R. The effective resistance between the two opposite vertices of the hexagon is :

Options:

A. R

B. 2R

C. $\frac{3R}{2}$

D. $\frac{2R}{3}$

E. 3R

Answer: C

Solution:

Solution:

Question 48

Two metallic solid spheres A and B, have radius R and 3R, respectively. The solid spheres are charged and kept isolated. Then, the two spheres are connected to each other through a thin conducting wire. The ratio of the final charge on the spheres A to B is :

Options:

A. 1 : 1

B. 1 : 3

C. 3 : 1

D. 1 : 9 E. 9

: 1

Answer: B

Solution:

Solution:

Question 49

A heat engine operates between a cold reservoir and a hot reservoir. The engine takes 200J of heat from the hot reservoir and has the efficiency of 0.4 . The amount of heat delivered to the cold reservoir in a cycle is :

Options:

A. 100J

B. 120J

C. 140J

D. 160J

E. 80J

Answer: B

Solution:

Solution:

Question 50

A system of ideal gas undergoes a thermodynamic process in which the initial pressure and volume are equal to the final pressure and volume. Let Δ Q is the heat supplied to the system, Δ W is the work done by the system and ΔU is the change in internal energy. The correct option is :

Options:

A. $\Delta Q = \Delta W$

B. $\Delta U > 0$

C. $\Delta U \neq 0$

D. $\Delta U + \Delta Q + \Delta W = 0$

E. $\Delta Q + \Delta W = 0$

Answer: A

Solution:

Solution:

Question 51

The rms speed of a gas having diatomic molecules at temperature T (in Kelvin) is 200m / s. If the temperature is increased to 4T and the molecules dissociate into monoatomic atoms, the rms speed will become :

Options:

A. 400m / s

B. 200m / s

C. 800m / s

D. $200\sqrt{2}$m / s

E. $400\sqrt{2}$m / s

Answer: E

Solution:

Solution:

Question 52

A metallic bullet with an initial velocity of 500m / s penetrates a solid object and melts. The initial temperature of the bullet is 30◦C and its melting point is 280◦C. The ratio of total heat generated to the initial kinetic energy of the bullet will be : [Latent heat of fusion of metal $= 3.0 \times 104J / kg$ and specific heat capacity of metal $= 200J / kg - K$]

Options:

A. 0.5

B. 1.0

C. 0.81

D. 0.36

E. 0.64

Answer: E

Solution:

Solution:

Question 53

Identify which type of electromagnetic wave is produced using Klystron or Magnetron valve :

Options:

A. Gamma rays

B. Microwave

C. Infrared rays

D. Ultraviolet rays

E. X-rays

Answer: B

Solution:

Solution:

Question 54

A long wire carrying a current of 5A lies along the positive z-axis. The magnetic field at the point with position vector $\vec{r}=(\hat{i}+2\hat{j}+2\hat{k})$ m will be :($\mu_0 = 4\pi\times10^{-7}$ in SI units)

Options:

A. $-2\sqrt{5} \times 10^{-7}$ T

B. 5×10^{-7} T

C. 0.33×10^{-7} T

D. -0.66×10^{-7} T

E. $7\sqrt{5} \times 10^{-7}$ T

Solution:

Answer: A

Solution:

Question 55

Which of the following scientific principle is used to produce the ultra-high magnetic fields?

Options:

A. Magnetic confinement of plasma

B. Faraday's laws of electromagnetic induction

C. Controlled nuclear fusion

D. Motion of charged particles in electromagnetic fields

E. Superconductivity

Answer: E

Solution:

Solution:

Question 56

A laser beam with an energy flux of 20W/ 2cm is incident on a non-reflecting surface at normal incidence. If the surface has an area of $30cm^2$, the total momentum delivered by the laser in 30 minutes for complete absorption will be :

Options:

A. 2.8×10^{-3} kgm/ s

B. 4.2×10^{-3} kgm/ s

C. 3.6×10^{-3} kgm/ s

D. 3.3×10^{-3} kgm/ s

E. 2.4×10^{-3} kgm/ s

Answer: C

Solution:

Solution:

Question 57

A series LCR circuit consists of a variable capacitor connected to an inductor of inductance 50 mH, resistor of resistance 100Ω and an AC source of angular frequency 500 rad / s. The value of capacitance so that maximum current may be drawn into the circuit is :

Options:

A. 60μF B. 50μF C. 100μF D. 80μF E. 25μF

Answer: D

Solution:

Solution:

Question 58

A magnetic field of $(10\,\hat{k})$T exerts a force of $(4\hat{i}-3\hat{j})\times 10^{-12}$N on a particle having a charge of 10^{-9}C. The speed of the particle is :

Options:

A. 40m / s

B. $40\sqrt{2}$m / s

C. 50m / s

D. $50\sqrt{3}$m / s

E. $100\sqrt{2}$m / s

Answer: C

Solution:

Solution:

Question 59

A simple pendulum experiment is performed for the value of ' g ', the acceleration due to the Earth's gravity. The measured value of length of the pendulum is 25 cm with an accuracy of 1 mm and the measured time

for 100 oscillations is found to be 100 sec with an accuracy of 1sec. The percentage uncertainty in the determination of ' g ' is :

Options:

A. 9.8 B. 0.98 C. 4.8 D. 2.4 E. 1.4

Answer: D

Solution:

Solution:

Question 60

A combination of two charges +1 nC and −1 nC are separated by a distance of 1μm. This constituted electric dipole is placed in an electric field of 1000Vm^{-1} at an angle of 45. The torque and the potential energy on the electric dipole are :

Options:

A. $\frac{1}{\sqrt{2}} \times 10^{-12}$ N.m and $\frac{1}{\sqrt{2}} \times 10^{-12}$J

B. $\frac{1}{\sqrt{2}} \times 10^{-12}$ N.m and $\sqrt{2} \times 10^{-12}$J

C. $\sqrt{2} \times 10^{-12}$ N.m and $\frac{1}{\sqrt{2}} \times 10^{-12}$J

D. $\sqrt{2} \times 10^{-12}$ N.m and $\sqrt{2} \times 10^{-12}$J

E. $\frac{\sqrt{3}}{2} \times 10^{-12}$ N.m and $\frac{\sqrt{3}}{2} \times 10^{-12}$J

Answer: A

Solution:

Solution:

Question 61

In a current carrying coil of inductance 60 mH, the current is changed

from 2.5A in one direction to 2.5A in the opposite direction in 0.10 sec. The average induced EMF in the coil will be :

Options:

A. 1.2V B. 2.4V C. 3.0V D. 1.8V E. 0.6V

Answer: C

Solution:

Solution:

Question 62

An inductor coil with an internal resistance of 50Ω stores magnetic field energy of 180 mJ and dissipates energy as heat at the rate of 200W when a constant current is passed through it. The inductance of the coil will be :

Options:

A. 90 mH

B. 120 mH

C. 45 mH

D. 30 mH

E. 60 mH

Answer: A

Solution:

Solution:

Question 63

A current carrying long solenoid is formed by winding 200 turns per cm. If the number of turns per cm is increased to 201 keeping the current constant, then the magnetic field inside the solenoid will change by :

Options:

A. 0.2%

B. 0.4%

C. 0.5%

D. 1%

E. 2%

Answer: C

Solution:

Solution:

Question 64

A metallic cylindrical wire ' A' has length 10 cm and radius 3 mm. Another hollow cylindrical wire ' B ' of the same metal has length 10 cm, inner radius 3 mm and outer radius 4 mm. The ratio of the resistance of the wires A to B is :

Options:

A. $\frac{7}{9}$

B. $\frac{9}{7}$

C. $\frac{9}{16}$

D. $\frac{16}{9}$

E. $\frac{3}{4}$

Answer: A

Solution:

Solution:

Question 65

A small bar magnet lies along the x-axis with its centre fixed at the origin. If the m agnetic field at point $(5\hat{i})$m due to this magnet 4×10^{-6}T, then the magnetic field(at point $10\hat{j}$)m will b

Options:

A. 2.5×10^{-7}T

B 2×10^{-6}T

C. 1×10^{-6}T

D. 2.0×10^{-7}T

E. 8.0×10^{-8}T

Answer: A

Solution:

Solution:

Question 66

An ideal gas is compressed in volume by a factor of 2 , while keeping its temperature constant. The speed of sound in it is :

Options:

A. doubled

B. unchanged

C. reduced to half

D. increased by 4 times

E. reduced by 4 times

Answer: B

Solution:

Solution:

Question 67

In the magnetic meridian of a certain plane, the horizontal component of earth's magnetic field is 0.36 Gauss and the dip angle is 60◦. The magnetic field of the earth at this location is :

Options:

A. 0.72 Gauss

B. 0.18 Gauss

C. 0.42 Gauss

D. 0.56 Gauss

E. 0.81 Gauss

Answer: A

Solution:

Solution:

Question 68

A resistance R is connected across an ideal battery. The total power dissipated in the circuit is P. If another resistance R is added in series, the new total dissipated power is :

Options:

A. 2P

B. 4P

C. P

D. $\frac{P}{2}$

E. $\frac{P}{4}$

Answer: D

Solution:

Solution:

Question 69

A toroid with 500 turns of wire carries a current of (2π) Ampere. A metal ring inside the toroid provides the core and has susceptibility of 2 × 10−5. If the magnetization is 5 × 10−2A / m, then radius of the ring is :

Options:

A. 50 cm

B. 20π cm

C. $\frac{50}{\pi}$cm

D. 20 cm

E. 60 cm

Answer: D

Solution:

Solution:

Question 70

When a vibrating tuning fork moves towards a stationary observer with a speed of 50m / s, the observer hears a frequency of 350 Hz. The frequency of vibration of the fork is : (Take speed of sound = 350m / s)

Options:

A. 350 Hz

B. 400 Hz

C. 200 Hz

D. 300 Hz

E. 250 Hz

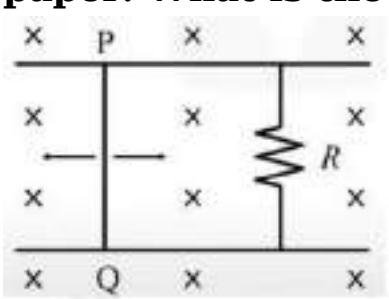
Answer: D

Solution:

Solution:

Question 71

The rod PQ slides along 2 parallel rails as shown in the figure. It has a length of 20 cm and is perpendicular to the 2 rails. It performs simple harmonic motion with amplitude 5 cm and frequency 10 Hz. The magnetic field is 10− 4T and is directed perpendicular to the plane of paper. What is the peak induced electromagnetic force?



Options:

A. $2\pi \times 10^{-7}$V

B. $4\pi^2 \times 10^{-3}$V

C. $2\pi \times 10^{-5}$V

D. $4\pi \times 10^{-5}$V

E. $\pi^2 \times 10^{-4}$V

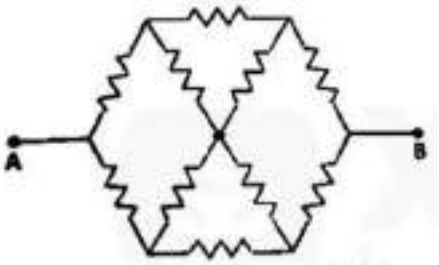
Answer: C

Solution:

Solution:

Question 72

Find the effective resistance between points A and B. Each resistance is equal to R.



Options:

A. 2R

B. $\frac{3}{4}$R

C. 3R

D. $\frac{4}{3}$R

E. $\frac{9}{5}$R

Answer: D

Solution:

Solution:

Question 73

The number of electrons in one mole of methane :

Options:

A. 6.023×10^{23}

B. 60.23×10^{23}

C. 0.6023×10^{23}

D. 602.3×10^{23}

E. 6023×10^{23}

Answer: B

Solution:

Solution:

Question 74

Which of the following statement cannot be explained by the proposals of Dalton's atomic theory ?

Options:

A. Reorganisation of atoms in chemical reactions

B. Identical propertics of all atoms of given element

C. The reason for combining of atoms

D. Formation of compounds from the combination of elements in a fixed ratio

E. Matter consists of individual atoms

Answer: C

Solution:

Solution:

Question 75

The correct order of variation of first ionisation enthalpies is :

Options:

A. Ne<Xe>Li>K<Cs

B. Xe<Li<K<Cs<Ne

C. Cs>K>Li>Xe>Ne

D. Li>K>Cs>Ne>Xe

E. Ne>Xe>Li>K>Cs

Answer: E

Solution:

Solution:

Question 76

Which of the following statements is WRONG?

Options:

A. The bond order of He_2 is zero; so He_2 molecule is unstable.

B. Li_2 molecule is diamagnetic.

C. O_2 molecule contains two unpaired electron and is paramagnetic.

D. C_2 molecule is paramagnetic in vapour phase.

E. H_2 molecule has no unpaired electrons,

Answer: D

Solution:

Solution:

Question 77

Find the WRONG statement from the following lists :

Options:

A. Dipole-dipole interaction exists in the HCl molecules.

B. Three states of matter are due to the balance between intermolecular forces and the thermal energy of the molecules.

C. According to kinetic theory of gases, the collisions of gas molecules are perfectly elastic.

D. Strength of hydrogen bond depends on the coulombic interaction between lone pair of electrons of one atom and the hydrogen atom.

E. Aqueous tension of water decreases with the increase in temperature.

Answer: E

Solution:

Solution:

Question 78

The hybridisation of Xe in XeF

Options:

2 is :

A. sp^3

B. sp^3d

C. sp^3d^2

D. sp^2d

E. Sp^2

Answer: B

Solution:

Solution:

Question 79

Which of the following compounds is known as inorganic benzene?

Options:

A. $B_3N_3H_6$

B. C_5H_5B

C. $C_3N_3H_3$

D. $B_3N_3H_6$

E. BF_3

Answer: D

Solution:

Solution:

Question 80

The number of S − S bonds and the number of lone pairs in S_8 molecule, respectively, are :

Options:

A. 8,8

B. 8,16

C. 16,8

D. 8,4

E. 4,8

Answer: B

Solution:

Solution:

Question 81

The shape of XeOF4 molecule is :

Options:

A. Square pyramid

B. Planar

C. Trigonal bipyramid

D. Pentagonal bipyramid

E. Linear

Answer: A

Solution:

Solution:

Question 82

The geometry of $[NiCl_4]^{2-}$ and $[Ni(CN)_4]^{2-}$ ions are :

Options:

A. Both tetrahedral

B. Both square planar

C. Both octahedral

D. Square planar and tetrahedral, respectively

E. Tetrahedral and square planar, respectively

Answer: E

Solution:

Solution:

Question 83

Which of the following compounds extensively has Mg as an important element in the living world ?

Options:

A. Haemoglobin

B. ATP

C. Florigen

D. Ferritin

E. Chlorophyll

Answer: E

Solution:

Solution:

Question 84

The basic character of the hydrides of 15 group elements decreases in the order :

Options:

A. $NH_3 > PH_3 > AsH_3 > SbH_3$

B. $SbH_3 > AsH_3 > PH_3 > NH_3$

C. $NH_3 > AsH_3 > PH_3 > SbH_3$

D. $NH_3 > SbH_3 > PH_3 > AsH_3$

E. $SbH_3 > PH_3 > AsH_3 > NH_3$

Answer: A

Solution:

Solution:

Question 85

Which of the following contains sp hybridised carbon atom?

Options:

A. $CH_3-CH=CH-CH_3$

B. $CH_3-C\equiv C-CH_3$

C. CH_3-CH_3

D. $CHCl_3$

E. CH_3-CH_2-Cl

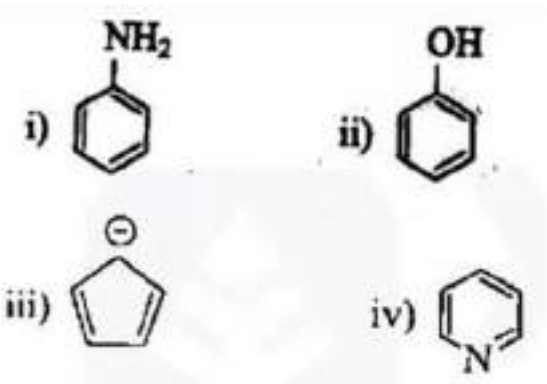
Answer: B

Solution:

Solution:

Question 86

Which are the non-benzenoid aromatic compounds in the following ?



Options:

A. iii and iv

B. i and iv

C. ii and iv

D. i and iv

E. ii and iii

Answer: A

Solution:

Solution:

Question 87

Which of the following is the most stable carbocation?

Options:

A. $CH_3 - \overset{\oplus}{C}H_2$

B. $\overset{\oplus}{C}H\ 3$

C. $CH_3 - \overset{\oplus}{C}H - CH_3$

D. $(CH_3)_3\overset{\oplus}{C}$

E. $CH_3 - CH_2 - \overset{\oplus}{C}H_2$

Answer: D

Solution:

Solution:

Question 88

Which of the following cannot act as a nucleophile?

Options:

A. $CH_3\overset{\ominus}{O}$

B. H_2O

C. CH_3NH_2

D. $(CH_3)_3\overset{\oplus}{C}$

E. $CH_3CH_2\overset{\ominus}{O}$

Answer: D

Solution:

Solution:

Question 89

What are the products of the following reactions?

i) $CH_3-CH_2-Br+Na \xrightarrow{\text{Dry ether}}$

ii) $CH_3COONa + NaOH \xrightarrow[\Delta]{CaO}$

Options:

A. i) CH_3-CH_3 and ii) $CH_2=CH_2$

B. i) $CH_3-CH_2-CH_3$ and ii) CH_3-CH_3

C. i) $CH_3-CH_2-CH_3$ and ii) CH_4

D. i) $CH_3-CH_2-CH_2-CH_3$ and ii) $HC\equiv C-H$

E. i) $CH_3-CH_2-CH_2-CH_3$ and ii) CH_4

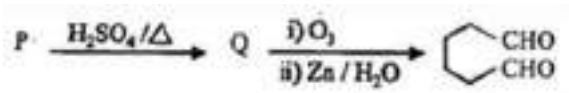
Answer: E

Solution:

Solution:

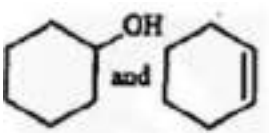
Question 90

Find the compounds P and Q in the following reactions :

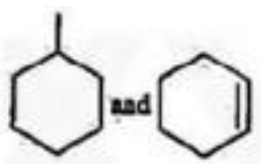


Options:

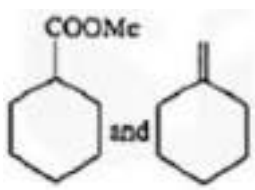
A.



B.



C.



The tetrahedral crystal field splitting is only of the octahedral splitting.

Options:

A. $\frac{1}{9}$

B. $\frac{2}{9}$

C. $\frac{3}{4}$

D. $\frac{9}{5}$

E. $\frac{9}{9}$

Answer: D

Solution:

Solution:

Question 93

Which order is correct in spectrochemical series of ligands :

Options:

A. $Cl^- < F^- < [CO_2O_4]^{2-} < H_2O < CN^-$

B. $Cl^- < F^- < CN^- \leq H_2O < [CO_2O_4]^{2-}$

C. $F^- < Cl^- < CN^- < H_2O < [CO_2O_4]^{2-}$

D. $F^- < Cl^- < H_2O < CN^- < [CO_2]^{2-}_4$

E. $Cl^- < F^- < H_2O < [CO_2O_4]^{2-} < CN^-$

Answer: A

Solution:

Solution:

Question 94

HF is a liquid unlike other hydrogen halides because :

Options:

A. $H-F$ bond is strong

B. Hydrogen bonding is present

C. HF is a weak acid

D. F atom is smaller in size

E. HF is a strong base

Answer: B

Solution:

Solution:

Question 95

The order of acidity follows :

Options:

A. HF > HCl > HBr > HI

B. HF > HBr > HCl > HI

C. HI > HCl > HF > HBr

D. HI > HBr > HCl > HF

E. HBr > HCl > HF > HI

Answer: D

Solution:

Solution:

Question 96

The correct order of O − O bond length in O_3, O_2 and H_2O_2 is :

Options:

A. $O_2 > HO_2 > O_3$

B. $O_3 > HO_2 > O_2$

C. $HO_2 > O > O_2$

D. $HO_2 > O > O_3$

E. $O_2 > O_3 > HO_2$

Answer: D

Solution:

Solution:

Question 97

Geometry, hybridisation and magnetic moment of $[MnBr_4]^{2-}$, $[FeF_6]^{4-}$ and $[Ni(CN)_4]^{2-}$ ions, respectively, are :

Options:

A. Tetrahedral, square planar, octahedral; sp^3, dsp^2, sp^3d^2; 5.9, 0, 4.9

B. Tetrahedral, octahedral, square planar; sp^3, sp^3d^2, dsp^2; 5.9, 4.9, 0

E. Tetrahedral, octahedral, square planar; sp^3, sp^3d^2, dsp^2; 5.9, 0, 4.9

. Octahedral, square planar, tetrahedral; sp^3d^2, dsp^2, sp^3; 4.9, 0, 5.9

. Square planar, tetrahedral, octahedral; sp^3d^2, sp^3, dsp^2; 0, 4.9, 5.9

Answer: B

Solution:

Solution:

Question 98

What is the probable ratio between the root mean square speed (rms), average speed (av) and the most probable speed (mp)?(U = speed of the gas molecules)

Options:

A. $U_{mp} : U_{rms} : U_{av}$: 1.128 : 1 : 1.224

B. $U_{av} : U_{rms} : U_{mp}$: 1 : 1.128 : 1.224

C. $U_{mp} : U_{av} : U_{rms}$: 1 : 1.128 : 1.224

D. $U_{mp} : U_{av} : U_{rms}$: : 1.224 : 1 : 1.128

E. $U_{rms} : U_{mp} : U_{av}$: : 1 : 1.128 : 1.224

Answer: C :

Solution:

Solution:

Question 99

Which is the WRONG statement from the following lists ?

Options:

A. No work is done during free expansion of an ideal gas for both reversible and irreversible processes.

B. The density and pressure are extensive properties but the enthalpy and heat capacity are intensive properties.

C. The change in enthalpy (Δ H) is negative for exothermic reactions but is positive for endothermic reactions.

D. The difference between change in enthalpy (ΔH) and the internal energy (ΔU) is not significant for solids and liquids, but significant for gases.

E. The standard enthalpy change of fusion of CH COCH is higher than that of N .

3 3 2

Answer: B

Solution:

Solution:

Question 100

The magnitude of equilibrium constant for the gaseous reaction of

H2(g) with I2(g) for the formation of 2 HI(g) is 57 at a particular temperature. The molar concentrations, [H2] = 0.10M, [I2] = 0.20M and [HI] = 0.40M are found to be at the same temperature. Find the CORRECT statement about the reaction :

Options:

A. The mixture of H (g), I (g) and HI(g) is at equilibrium.

2 2

B. More H_2(g) and I_2(g) will not react to form more HI(g).

C. The concentration of H_2(g) and I_2(g) will decrease till the equilibrium constant is equal to reaction quotient.

D. Reaction quotient is independent of concentration.

E. If reaction quotient is greater than equilibrium constant of the reaction, more HI(g) will be formed.

Answer: C

Solution:

Solution:

Question 101

The pKa of acetic acid is 4.76 . What will be the pKb of ammonium hydroxide, if the pH of ammonium acetate is 7.00 ?

Options:

A. 4.770 B. 4.765 C. 4.755 D. 4.750 E. 4.740

Answer: B

Solution:

Solution:

Question 102

In oligosaccharides, how many monosaccharides will be present?

Options:

A. 1 to 5

B. 2 to 10

C. 4 to 5

D. 1 to 15

E. 3 to 5

Answer: B

Solution:

Solution:

Question 103

In DNA molecule, the sugar part is…… and in RNA molecule, the

sugar part is.....

Options:

A. β − D − 2− ribose and α − L-ribose

B. β − D − 2-deoxy ribose and α-L-ribose

C. β − D − 3-deoxy ribose and α-D-ribose

D. α − D − 2-deoxy ribose and β − D-ribose

E. β − D − 2− deoxy ribose and β − D ribose

Answer: E

Solution:

Solution:

Question 104

Which statement is correct in the following?

Options:

A. Amylose is a polymer of α − D-glucose.

B. Amylose is a polymer of β − D-glucose.

C. Cellulose is a polymer of α − D− glucose.

D. Cellulose is a polymer of β − D galactose.

E. Amylose is a polymer of α - Dgalactose.

Answer: A

Solution:

Solution:

Question 105

Calculate the log of equilibrium constant (logK$_c$) in reaction,

$Mg(s) + 2Ag^+(aq) \rightarrow Mg^{2+}(aq) + 2Ag(s)$

Given that $E_{cell}^\circ = 3.245V$

Options:

A. 100.5

B. 110.5

C. 10

D. 100

E. 110

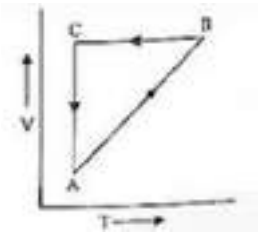
Answer: E

Solution:

Solution:

Question 106

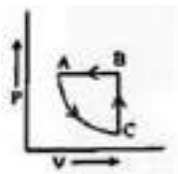
The following diagram shows the V − T diagram for a process ABCA



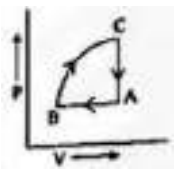
The corresponding P − V diagram is :

Options:

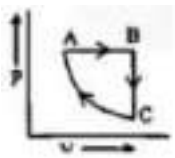
A.



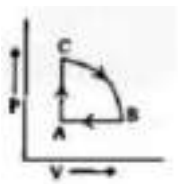
B.



C.



D.



E.

Answer: C

Solution:

Solution:

Question 107

In which of the following, entropy decreases?

Options:

A. Liquid water is converted to gas.

B. Liquid water crystallizes to ice.

C. $H_2(g) \rightarrow 2H(g)$

D. $NH_4Cl(s) \rightarrow NH_3(g) + HCl(g)$

E. Temperature of NaCl(s) raises from 298 to 517K.

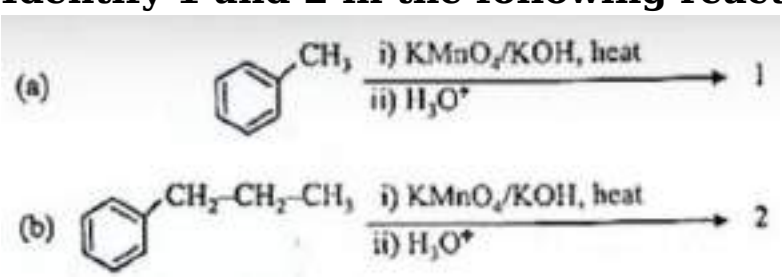
Answer: B

Solution:

Solution:

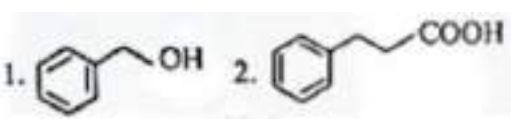
Question 108

Identify 1 and 2 in the following reactions:



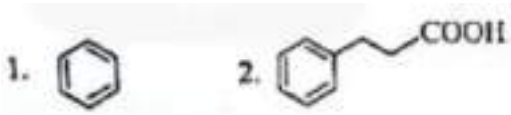
Options:

A.

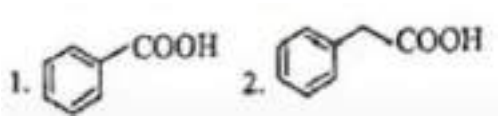


B.

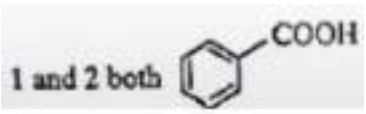
C.



D.



E.



Answer: E

Solution:

Solution:

Question 109

In which of the following reactions, we will get new C − C bond ?

Options:

A. Cannizzaro reaction and Aldol condensation reaction

B. Cannizzaro reaction and Sandmeyer's reaction

C. Friedel-Crafts reaction and GattermannKoch reaction

D. Cannizzaro reaction and ReimerTiemann reaction

E. Sandmeyer's reaction and Aldol condensation reaction

Answer: C

Solution:

Solution:

Question 110

The nitrogen oxide that does not contain N − N bond is :

Options:

A. N_2O_5

B. N_2O_3

C. NO_2

D. N_2O_4

E. N_2O

Answer: A

Solution:

Solution:

Question 111

In a zero-order reaction, the reactant A disappeared with a rate of reaction k = $0.04Msec^{-1}$. The initial concentration of A is 1M. What will be the concentration of A after 20 seconds?

Options:

A. 1.08M

B. 0.2M

C. 0.8M

D. 0.002M

E. 0.008M

Answer: B

Solution:

Solution:

Question 112

Following of which can be an empirical relationship between the quantity of gas adsorbed by unit mass of solid adsorbent and pressure at a particular temperature ?x = mass of the gas adsorbed on a mass ' m ' of the adsorbent at a pressure' P ∴ k and n are constants, which depend on the nature of the adsorbent and the gas at a particular teraperature.

Options:

A. $\log x + \log m = \log k + \frac{1}{n}\log P$

B. $\log x + \log m = \log k - \frac{1}{\pi}\log P$

C. $\log x + \log m = -\log k + \frac{1}{n}\log P$

D. $\log x - \log m = \log k + \frac{1}{n}\log P$

E. $\log x - \log m = \log k - \frac{1}{n}\log P$

Answer: D

Solution:

Solution:

Question 113

In the following which can be used as an antidepressant drug?

Options:

A. Salvarsan

B. Ofloxacin

C. Erythromycin

D. Serotonin

E. Chloroxylenol

Answer: D

Solution:

Solution:

Question 114

$[Co(NH_3)_4(NO_2)_2]Cl$ exhibits :

Options:

A. Linkage isomerism, ionisation isomerism and optical isomerism

B. Linkage isomerism, geometrical isomerism and ionisation isomerism

C. Ionisation isomerism, geometrical isomerism and optical isomerism

D. Linkage isomerism, geometrical isomerism and optical isomerism

E. Optical isomerism, geometrical isomerism and ionisation isomerism

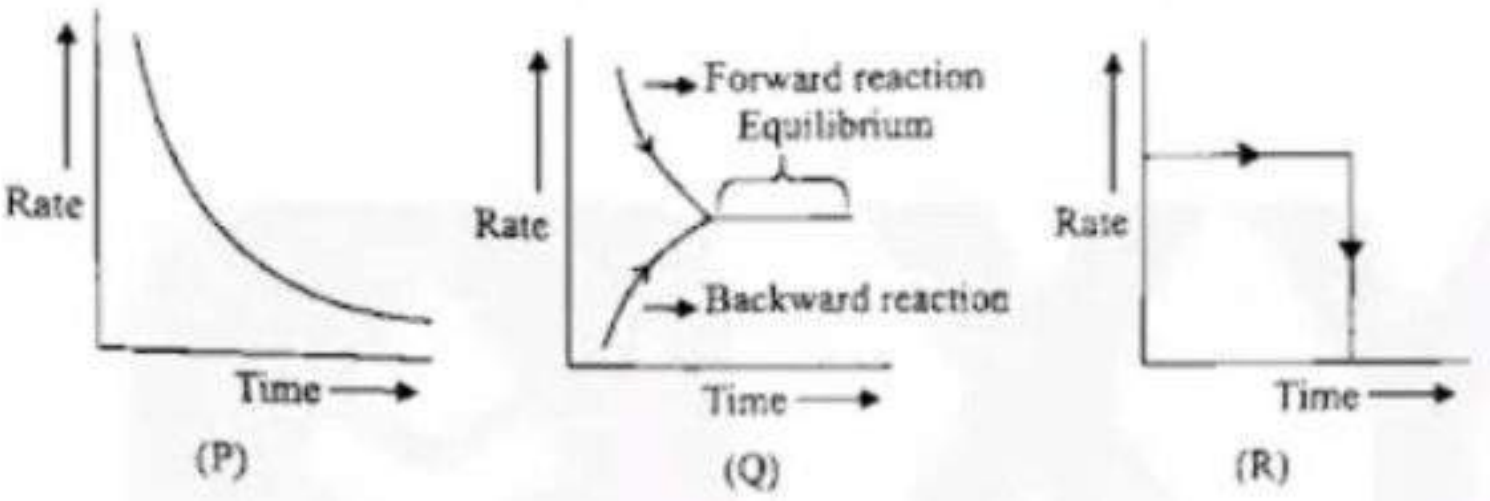
Answer: B

Solution:

Solution:

Question 115

Find the correct combination about the following plots (P, Q and R) for the variation of rate of reaction with time.



Options:

A. Q = Reversible; P = Zero order, R= Irreversible

B. R = Zero order, P = Zero order, R = Irreversible

C. Q = Irreversible; R = Reversible; P= Zero order

D. P = Irreversible; Q = Reversible; R = Zero order

E. P = Reversible; Q = Zero order, R = Irreversible

Answer: D

Solution:

Solution:

Question 116

The resistance of the cell containing the aqueous solution of NaCl at 20◦C is 60 ohm. If the specific conductivity of this solution at 20◦C is 0.04ohm− 1cm− 1, what is the cell constant in cm− 1 ?

Options:

A. 2.0 B. 1.5

C. 0.5

D. 0.15

E. 2.4

Answer: E

Solution:

Solution:

Question 117

Match the following columns (P) with (Q)

(P)	(Q)
a) Grignard reagent	(i) $AlCl_3$
b) Sandmeyer's reaction	(ii) Sodium netal
c) Cannizzaro reaction	(iii) Cu(I)
d) Friedel-Crafts reaction	(iv) $CH_3MgBr^{2}9$
e) Wurte reaction	(v) NaOH

Options:

A. a) - (iv); b)-(iii); c)-(ii); d) - (i); e)-(v)

B. a) - (v); b)-(ii); c)-(iii); d)-(iv); e)-(i)

C. a) - (iv); b)-(i); c)-(v); d) -(iii); e) - (ii)

D. a) - (ii); b) - (iii); c)-(i); d) - (v); e) - (iv)

E. a)-(iv); b)-(iii); c)-(v); d) -(i); e)-(ii)

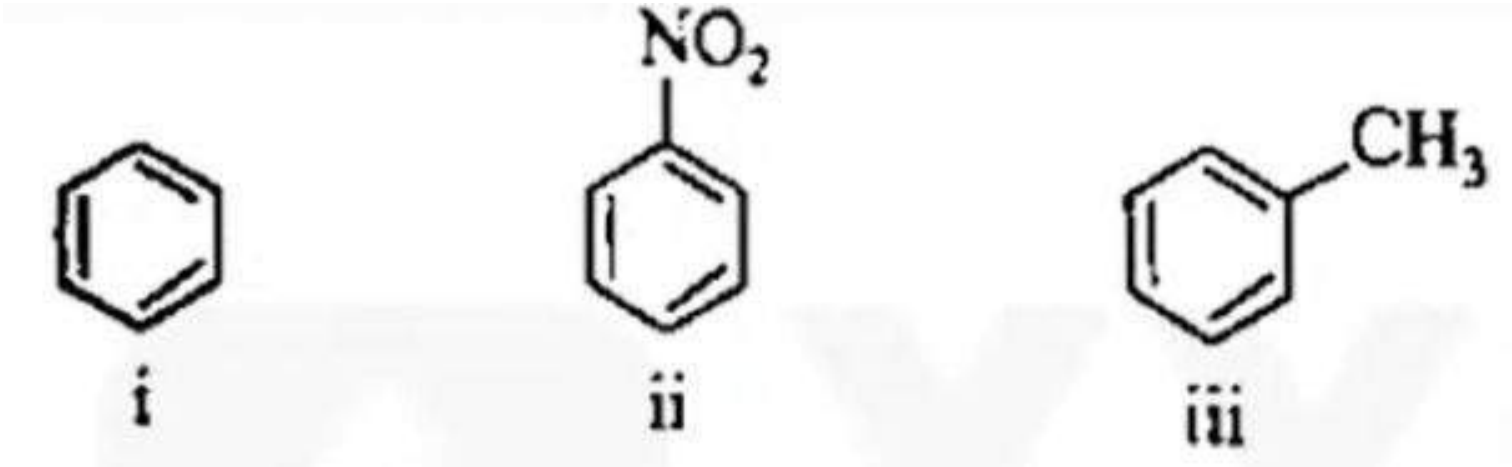
Answer: E

Solution:

Solution:

Question 118

Which compound will not take part in the Friedel-Crafts acylation?



Options:

A. ii and iii

B. only iii

C. i and iiii

D. only ii

E. only i

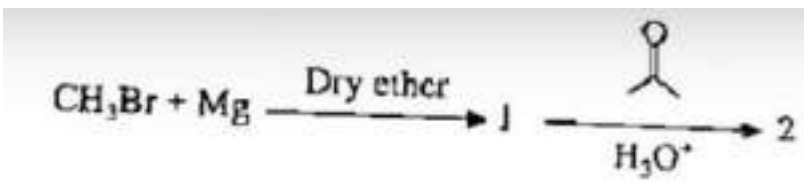
Answer: D

Solution:

Solution:

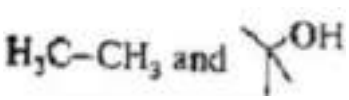
Question 119

Identify 1 and 2 in the following reaction

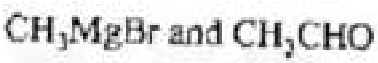


Options:

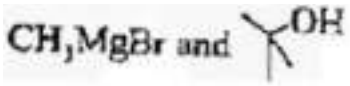
A.



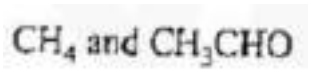
B.



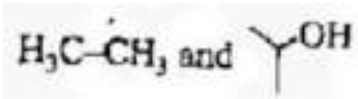
C.



D.



E.



Answer: C

Solution:

Solution:

Question 120

What is the major product in the following reaction?
CH3 − CH2 − CH2 − CH = CH2 + HBr→

Options:

A. $CH_3 - CH_2 - CH_2 - CH_2 - CH_2 - Br$

B. $CH_3 - CH_2 - \overset{\overset{Br}{|}}{CH_2 - CH} - CH_3$

C. $CH_3 - CH_2 - CH_2 - \overset{\overset{Br}{|}}{CH} - CH_3$

D. $CH_3 - CH_2 - CH_2 - \overset{\overset{Br}{|}}{CH} - CH_2 - Br$

E. $CH_3 - CH_2 - CH_2 - CH_2 - CH_3$

Answer: B

Solution:

Solution:

Maths

Question 1

Let $f : R \to R$ be a function defined by $f(x) = x^2 + 9$. The range of f is

Options:

A. R

B. $(-\infty, -9] \cup [9, \infty)$

C. $[9, \infty)$ D. $[3, \infty)$

E. $[3, \infty) \cup (-\infty, -3]$

Answer: C

Solution:

Solution:

$x^2 \geq 0$

$x^2 + 9 \geq 9$

$f(x) \geq 9$

$\Rightarrow [9, \infty)$

Question 2

Let $f(x) = \frac{x-1}{x+1}$. Let $S = \{ x \in \mathbb{R} \mid f \circ f^{-1}(x) = x \text{ . does not hold} \}$. The cardinality of S is

Options:

A. a finite number, but not equal to 1, 2, 3

B. 3 C. 2 D. 1 E. infinite

Answer: C

Solution:

Solution:

$f(x)$ is not defined for $x = -1$

$f^{-1}(x) = \frac{x+1}{x-1}$ is not defined for $x = 1$

$\therefore f \circ f^{-1}(x) = x$ does not hold at $x = \pm 1$

$n(s) = 2$

Question 3

The domain of the real valued function $f(x) = \sqrt{x^2 - 4} + \frac{1}{\sqrt{x^2 - 7x + 6}}$ is

Options:

A. $R - [-6, -2)$

B. $R - [-6, 2)$

C. $\mathbb{R} - [-2, 6)$

D. $R - (2, 6]$

E. $R - (-2, 6]$

Answer: E

Solution:

Solution:

$x^2 - 4 \geq 0 \Rightarrow x^2 \geq 4$

$\Rightarrow x \in (-\infty, -2] \cup [2, \infty)$ ... (1)

$x^2 - 7x + 6 > 0 \Rightarrow (x - 1)(x - 6) > 0$

$\Rightarrow x \in (-\infty, 1) \cup (6, \infty)$ ... (2)

Intersection of

(1) and (2) is $R - (-2, 6]$

Question 4

The number of solutions of the equation $\frac{1}{2}(x^3 + 1) = \sqrt[3]{2x - 1}$ is

Options:

A. 0 B. 6 C. 9 D. Infinite E. 3

Answer: E

Solution:

Solution:

RHS is inverse of LHS.

Equation is cubic

$\therefore$ there are 3 solutions

Question 5

Let a, b, c, d be an increasing sequence of real numbers, which are in

geometric progression. If a + d = 112 and b + c = 48, then the value of $\frac{a+c+8}{b}$ is

Options:

A. 1

B. 5

C. 4

D. 3

E. 2

Answer: C

Solution:

Solution:

Let $a = a, b = ar, c = ar^2, d = ar^3$

Given $a + ar^3 = 112$

$ar + ar^2 = 48$

$ar(1 + r) = 48$

$\therefore \frac{a(1+r^3)}{ar(1+r)} = \frac{112}{48} = \frac{7}{3}$

$\Rightarrow 1 - r + r^2 = \frac{7r}{3}$

$\Rightarrow 3r^2 - 10r + 3 = 0$

$r = \frac{10 \pm \sqrt{100-36}}{6} = 3, \frac{1}{3}$

Since GP is increasing $r \neq \frac{1}{3} < 1$

$\therefore r = 3 \Rightarrow a = \frac{112}{28} = 4$

$\frac{a+c+8}{b} = \frac{10a+8}{3a} = \frac{48}{12} = 4$

Question 6

Let a, b be two real numbers between 3 and 81 such that the resulting sequence 3, a, b, 81 is in a geometric progression. The value of a + b is

Options:

A. 29 B. 90 C. 27 D. 81 E. 36

Answer: E

Solution:

Solution:

$a = 3r$

$b = 3r^2$

$c = 3r^3 = 81 \Rightarrow r = 3$

$a + b + c = (3 + 9 + 27) = 39 = r \quad 36$

Question 7

Let a1, a2, a3, ... be an increasing sequence of natural numbers, which are in an arithmetic progression with common difference d . Suppose $a_1 + a_2 + a_3 = 27$ and $a_1^2 + a_2^2 + a_3^2 = 275$. Then the values of a1,d are

Options:

A. $a_1 = 3; d = 2$

B. $a_1 = -5; d = 4$

C. $a_1 = 4; d = 5$

D. $a_1 = -4; d = 5$

(F) $a_1 = 5; d = 4$

Answer: D

Solution:

Solution:

$a1 = a$

$a2 = a + d$

$a3 = a + 2d$

$3a + 3d = 27$ (given)

$a + d = 9$

$a_1^2 + a_2^2 + a_3^2 = 3a^2 + 6ad + 5d^2$

$275 = [3a^2 + 2ad + a^2] + 2d^2$

$= 3(a + d)^2 + 2d^2$

$275 = 3(81) + 2d^2$

$d^2 = \frac{275 - 243}{2} = 16$

$\therefore d = a + 4 = 9 \Rightarrow a = 5$

$a = 5, d = 4$

Question 8

The sides of a right-angled triangle are in an arithmetic progression. If the area of the triangle is 54 , then the length of the longest side is

Options:

A. 6 B. 12 C. 15

D. 9

E. 18

Answer: C

Solution:

Solution:

Let $a-d, a, a+d$ be the sides. By Pythagoras theorem

$a^2 + (a-d)^2 = (a+d)^2$

$a^2 + a^2 - 2ad + d^2 = a^2 + 2ad + d^2$

$a^2 = 4ad$

$a = 4d$

$\text{area} = \frac{1}{2}a(a-d) = 54$

$\frac{1}{2} \cdot 4d \cdot 3d = 54$

$d^2 = \frac{54}{6} = 9$

$d = 3, a = 4d = 12$, (length is positive and so we avoid $d = -3$)

$a + d = 15$

Question 9

Let A be (2n + 1) × (2n + 1) matrix with integer entries and positive determinant. Where $n \in \mathbb{N}$. If $AA^{T} = I = A^{T}A$ then which of the following statements always holds ?

Options:

A. $\det(A) = 0$

B. $\det(A) \neq 0$

C. $\det(A + I) \neq 0$

D. $\det(A) \neq 0$

E. $\det(A + I) = 0$

Answer: D

Solution:

Solution:

A is orthogonal matrix of odd order Property $\det(A-I)=0$

Question 10

The inequality $\frac{2x-1}{3} \geq \frac{3x-2}{4} - \frac{(2-x)}{5}$ holds for x belonging to

Options:

A. $\mathbb{R}$

B. $(-\infty, 3]$

C. $(-\infty, -3] \cup [3, \infty)$

D. $(-\infty, 2]$

E. $(-\infty, 2] \cup [4, \infty)$

Answer: D

Solution:

Solution:

$\frac{2x-1}{3} \geq \frac{19x-18}{20}$

$40x - 20 \geq 57x - 54$

$\Rightarrow x \leq \frac{34}{17} = 2$

$(-\infty, 2]$

Question 11

The contrapositive of the statement "If the number is not divisible by 3 , then it is not divisible by 15˚ is

Options:

A. If the number is not divisible by 3 , then it is not divisible by 15

B. If the number is not divisible by 15 , then it is not divisible by 3

C. If the number is not divisible by 15 , then it is divisible by 3

D. If the number is divisible by 15 , then it is divisible by 3

E. If the number is divisible by 15 , then it is not divisible by 3

Answer: D

Solution:

Solution:

$p \rightarrow q$

Contrapositive $\sim q \rightarrow \sim p$

Question 12

Let A be an invertible matrix of size 4 × 4 with complex entries. If the determinant of of adj (A) is 5. then the number of possible value of determinant of A is

Options:

A. 1 B. 4

C. 6 D. 3

E. 2

Answer: D

Solution:

Solution:

A is invertible $\Rightarrow |A| \neq 0$

$|adjA| = |A|^{n-1}$

$= |A|^{4-1}$

$= |A|^3 =$

$|A| = \sqrt[3]{5} =$ there are 3 roots

Question 13

The determinant of the matrix

$$\begin{bmatrix} 1 & 4 & 8 \\ 1 & 9 & 2 \\ 7 & 1 & 16 \end{bmatrix}$$ **is**

Options:

A. 13 B. 208

C. 104

D. 26 E. 52

Answer: E

Solution:

Solution:

$$\begin{vmatrix} 1 & 4 & 8 \\ 1 & 9 & 2 \\ 7 & 1 & 16 \end{vmatrix}$$

$$= \begin{vmatrix} 8 & 4 \\ 0 & 5 \end{vmatrix}$$

$$\begin{vmatrix} 9 & 5 \\ 12 & 56 \end{vmatrix}$$

$= (56 \cdot 5 - 12 \cdot 19) = 280 - 228 = 52$

Question 14

If A= $\begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A = A^T$, then which of the following statements is true

Options:

A. $5a - b = -5$

B. $5a + b = 10$

C. $\det(A) < 0$

D. A is symmetric

E. $\det(A) \geq 0$

Answer: E

Solution:

Solution:

$A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$

$AadjA = AA^T$

$A^{-1}AadjA = A^{-1}AA^T$

$adjA = A^T$

$\begin{bmatrix} 2 & b \\ -3 & 5a \end{bmatrix} = \begin{bmatrix} 5a & 3 \\ - & \end{bmatrix}$

$\Rightarrow b = 3, a = \frac{2}{5}, 5a - b = 2 - 3 = -1 \neq -5$

$5a + b = 5 \neq 10$

$\therefore A = \begin{bmatrix} 2 & -3 \\ 3 & 2 \end{bmatrix}$ .A is not symmetric

$|A| = 4 + 9 = 13$

$\therefore \det(A) \geq 0$

Question 15

Suppose A = $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ is an adjoint of the matrix $\begin{bmatrix} 1 & 3 \\ 3 & \\ 1 & 4 \\ 3 & \\ 1 & 3 \\ 4 & \end{bmatrix}$. The value of $\frac{a_1 + b2 + }{b_1 a_2}$ is

Options:

A. 0 B. 3

C. 1 D. 2

E. 4

Answer: B

Solution:

Solution:

a_1 = cofactor of $a_{11} = 4-3=1$

b_2 = cofactor of $a_{22} = 4-3=1$

c_3 = cofactor of $a_{33} = 4-3=1$

b_1 = cofactor of $a_{12} = -1$

a_2 = cofactor of $a_{21} = -3$

$\therefore \frac{a_1 + b_2 + c_3}{b_1 a_2} = \frac{7+1+1}{3} = \frac{9}{3} = 3$

Question 16

If $x + iy = \frac{1}{(1+\cos\theta) + i\sin\theta}$, then the value or $x^2 + 1$ is

Options:

A. $\frac{7}{4}$

B. $\frac{13}{4}$

C. $\frac{1}{4}$

D. $\frac{9}{4}$

E. $\frac{5}{4}$

Answer: E

Solution:

Solution:

$x + iy = \frac{1}{(1+\cos\theta) + i\sin\theta}$

$= \frac{(1+\cos\theta) - i\sin\theta}{(1+\cos\theta)^2 + \sin^2\theta}$

Real part $x = \frac{1+\cos\theta}{1+2\cos\theta+1}$

$= \frac{(1+\cos\theta)}{2(1+\cos\theta)} = \frac{1}{2}$

$x^2 + 1 = \frac{1}{4} + 1 = \frac{5}{4}$

Question 17

If α, β, γ are the cube roots of -2 , then the value of $\frac{x\alpha + y\beta + z\gamma}{x\beta + y\gamma + z\alpha}$ is (x, y, z are variables)

Options:

A. $e^{i\pi / 3}$

B. $e^{2\pi i / 3}$

C. 1

D. -1

E. $e^{4\pi i/\ 3}$

Answer: E

Solution:

Solution:

$\sqrt[3]{-1} = -1, -\omega, -\omega^2$

$\sqrt[3]{-2} = -2, -2\omega, -2\omega^2$

α, β, γ

$$\frac{x(-2) + y(-2\omega) + z(-2\omega^2)}{x(-2\omega) + y(-2\omega^2) + z(-2)}$$

$$= \frac{x + y\omega + z\omega^2}{x\omega + y\omega^2 + z\omega^3} = \frac{x + y\omega + z\omega^2}{\omega(x + y\omega + z\omega^2)}$$

$$= \frac{1}{\omega} = \frac{\omega^2}{3\omega} = \omega$$

$$= \frac{-1}{2} - \frac{i\sqrt{3}}{2} \text{ lies in 3}^{rd} \text{ quadrant}$$

$$\therefore e^{\frac{4\pi}{3}i}$$

Question 18

Let $x + \frac{1}{x} = 2\cos\alpha$. For any $n \in \mathbb{N}$, the value of $x^n - \frac{1}{x^n}$ is

Options:

A. $\cos(n\alpha)$

B. $2\cos(n\alpha)$

C. $2i\sin(n\alpha)$

D. $i\sin(n\alpha)$

E. $4\cos(n\alpha)$

Answer: C

Solution:

Solution:

$x + \frac{1}{x} = 2\cos\alpha$

$x^2 - 2\cos\alpha x + 1 = 0$

$x = \cos\alpha + i\sin\alpha$

Using De Moivre's theorem

$x^n = (\cos\alpha + i\sin\alpha)^n = \cos n\alpha + i\sin n\alpha$

$\frac{1}{x^n} = \cos n\alpha - i\sin n\alpha$

$x^n - \frac{1}{x^n} = 2i\sin n\alpha$

Question 19

If $f(z) = z^n + a_{n-1}z^{n-1} + \cdots + a_1 z + a_0 \in \mathbb{R}[z]$ is a polynomial in z with no root over $\mathbb{R}$. then deg(f) is

Options:

A. 9

B. always ≤ 4

C. an odd number

D. always ≥ 4

E. an even number

Answer: E

Solution:

Solution:

$f(z) = z^n + a_{n-1}z^{n-1} + \ldots + a_1 z + a_0$ has no real root

We know that complex roots occur in conjugate pairs

Consider $f(z) = 1 + z = 0$

$f(z) = 1 + z + z = 0$ and

$f(s) = 1 - 2 + 2 - 2 = 0$

$1 + z = 0$ has one real root

$1 - z + z^2 - z^3 = 0$ has real solution $z = 1$

$1 + z + z^2 = 0$ has two imaginary roots

ω, ω^2. From this we shall conclude that

to get imaginary roots the degree of $f(z)$ must be even

Question 20

Let $S = \{n \in \mathbb{N} \mid n^3 + 3n^2 + 5n + 3$. is not divisible by 3}. Then. which of the following statements is true about S

Options:

A. S = φ

B. |S| ≥ 2 and |S| is a multiple of 5

C. S is non-empty but |S| is finite

D. |S| is infinite

E. S is non-empty and |S| is a multiple of 3

Answer: A

Solution:

Solution:

$n^3 + 3n^2 + 5n + 3 = (n+1)(n+2n+3)$

$= (n+1)(n+1)+2]$

Consider the following cases

$n = 3k, n^3 + 3n^2 + 5n + 3$

$= (3k+1)[(3k+1)^2 + 2]$

$= (3k+1)[9k^2 + 6k + 3]$, which is divisible by 3

$n = 3k+1$,

$n^3 + 23n + 5n + 3$

$= (3k+2)(9k^2 + 12k + 4 + 2)$,

$= (3k+2)(9k^2 + 12k + 4 + 2)$, which is divisible by 3

$n = 3k+2$

$n^3 + 3n^2 + 5n + 3 = (3k+3)(3k+3) + 2^2$

which is divisible by 3

In all the above cases

$n^3 + 3n^2 + 5n + 3$ is divisible by 3

$\therefore n^3 + 3n^2 + 3n + 3$ is divisible by 3 for all $n \in Z$

$\therefore S = \phi$

Question 21

If the coefficients of $5r+)^{th}$ term and $(r-1)^{th}$ term in the expansion of $(1+)^{25}$ are equal, then r is

Options:

A. 6 B. 3

C. 5 D. 2

E. 4

Answer: E

Solution:

Solution:

$^{25}C_{5r+3} = {}^{25}C_{r-2}$

$\Rightarrow 5r + 3 + r - 2 = 25$

$6r = 24$

$r = 4$

Question 22

$\frac{\sum_{r=0}^{n}(4r+3)\cdot(C_r^n)^2}{(2n+3)}$ **is**

For any n ≥ 0, the value of is

Options:

A. $^{2n}C_{n-1}$

B. $^{8n}C_n$

C. $^{2n}C_{n+1}$

D. $^{n}C_{n-2}$

E. $^{2n}C_n$

Answer: E

Solution:

Solution:

$\sum_{r=0}^{n}\frac{(4r+3)(C_r^n)^2}{2n+3}$

Substitute for n = 1

$\frac{3+(C_0^1)^2+7(C_1^1)^2}{5} = \frac{10}{5} = 2$

$=2\cdot 1C_1 = 2C_1$

In the given choices option E matches with this

Question 23

The number of ways in which we can distribute n identical balls in k boxes is

Options:

A. $^{n}C_k$

B. $^{n}C_{(k-1)}$

C. $^{(n-k-1)}C_{(k-1)}$

D. $^{(n-1)}C_{(k-1)}$

E. $^{(n-k)}C_n$

Answer: C

Solution:

Solution:
Since blank boxes can be allowed $^{(n+k-1)}\ C_{(k-\ 1)}$ is the required number

---

Question 24

Suppose there are 5 alike dogs, 6 alike monkeys and 7 alike horses. The number of ways of selecting one or more animals from these is

Options:

A. 362 B. 363 C. 336 D. 335 E. 337

Answer: D

Solution:

Solution:
Alike animals no: of ways
5 Dogs 6 ways(including)
6 Monkeys 7 ways
7 Horses 8 ways
Required number $= 6 \cdot 7 \cdot 8 - 1 = 335$
Here 1 is the count of no selection

Question 25

Consider the following Linear Programming Problem (LPP) :

Maximize $Z = 60x1 + 50x2$

subject to

$x1+2x2 \leq 40$

$3x_1+2x2 \leq 60$

$x1,x2 \geq 0$

Then, the

Options:

A. LPP has a unique optimal solution.

B. LPP is infeasible.

C. LPP is unbounded.

D. LPP has multiple optimal solutions.

E. LPP has no solution.

Answer: A

Solution:

Solution:

z = 60x_1 + 50x_2

(0, 0) 0

(20,0)1200

(10,15)1350

(0,20)1000

∴ optimum at (10, 15)

z = 1350

Question 26

Consider the linear programming problem :

Minimize 3x1 + 4x2 + 2x3

subject to

x1+x2+x3≤6

x1+2x2+x3≤10

x1, x2, x3 ≥ 0

x

x

Then, the number of basic solutions are

Options:

A. 7 B. 9

C. 10 D. 8

E. 3

Answer: E

Solution:

Solution:

Minimize 3x1+4x2+ 2x_3

Subject to x_1+x2+3x ≥ 6

x1+2x2+x3≥10

1+x2+x3≥0

x

Dual

Maximize z = 6y1 + 10y2

Subject to y1+y2≤3

y1+2y2≤4

1+y2≥2

y1+y2≥0

y

y

Three basicsolutions

Question 27

In a linear programming problem, the restrictions under which the objective function is to be optimised are called as

Options:

A. decision variables B. objective function C. constraints D. integer solutions

E. optimal solutions

Answer: C

Solution:

Solution:

Question 28

Which of the following is the correct formulation of linear programming problem

Options:

A. Max Z

B. Max Z $= 2x_1 + x_2$; subject to $x_1 + x_2 \leq 10; x_1 \leq 3; x_1 \geq 0; x_2 \leq 0$

C. Min Z $= 3x_1 + 2x_2$; subject to $x_1 + 2x_2 \geq 11; 3x_1 + x_2 \geq 24; x_1, x_2 \leq 0$

D. Min Z $= x_1 + 5x_2$; subject to $2x_1 + 5x_2 \leq 10; x_1 + 3x_2 \leq 9; x_1, x_2 \geq 0$

E. Max Z $= 4x_1 + 3x_2$; subject to $x_1 + 9x_2 \geq 8; 2x_1 + 5x_2 \leq 9; x_1 \leq 0, x_2 \geq 0$

$= 2x_1 + 5x_2$: subject to $4x_1 + 9x_2 \leq 8; 2x_1 + 3x_2 \leq 9; x_1, x_2 \leq 0$

Answer: C

Solution:

Solution:
$x_1, x_2 \geq 0$ is mandatory

Question 29

Let A and B be two independent events such that the odds in favour of A and B are 1 : 1 and 3 : 2, respectively. Then the probability that only one

of the two occurs is

Options:

A. 0.6 B. 0.7 C. 0.8 D. 0.5

E. 0.4

Answer: D

Solution:

Solution:

$P(A) = \frac{1}{2}$, $P(A') = \frac{1}{2}$

$P(B) = \frac{3}{5}$, $P(B') = \frac{2}{5}$

$P(AB') + P(A'B) = \frac{2}{10} + \frac{3}{10}$

$= \frac{5}{10} = 0.5$

Question 30

A six faced fair die is rolled for a large number of times. Then, the mean value of the outcomes is

Options:

A. 4.5 B. 2.5 C. 3.5 D. 1.5 E. 3

Answer: C

Solution:

Solution:

Mean value = $\frac{n[1 + 2 + 3 + 4 + 5 + 6]}{6n}$

$= \frac{21}{6} = 3.5$

Question 31

Let the probability distribution of random variable X be

X	-2	-1	1	2	3
P(X = x)	k	$2k$	$2k$	k	$3k$

Then, the value of E (X 2) is

Options:

A. $\frac{19}{9}$

B. $\frac{13}{35}$

C. $\frac{3}{11}$

D. $\frac{9}{3}$

E. $\frac{7}{3}$

Answer: B

Solution:

Solution:

$\sum P(x) = 1$

$9k = 1$

$k = \frac{1}{9}$

$\sum (x^2) = \sum x^2 P(x) =$

$= \frac{4 + 2 + 2 + 4 + 27}{9} = \frac{13}{3}$

Question 32

Let the standard deviation of x_1, x_2 and x_3 be 9 . Then, the variance of $3x_1+4$, $3x_2+4$ and $3x_3+4$ is

Options:

A. 243

B. 81

C. 729

D. 9

E. 733

Answer: C

Solution:

Solution:
Var(ax + b) = a^2var(x)
var(3x + 9) = 3^2var(x)
= 9 var(x)
= 9σ^2
= 9 81 = 729

Question 33

If the median of the observations 4, 6, 7, x, x + 2, 12, 12, 13 arranged in an increasing order is 9 , then the variance of these observations is

Options:

A. $\frac{37}{4}$

B. $\frac{38}{4}$

C. 8

D. 9

E. 10

Answer: A

Solution:

Solution:
$\frac{x + (x+2)}{2} = 9$
$x = \frac{18}{2}$
$\sigma^2 = \frac{E(x - \bar{x})^2}{n}$

x	$(x-\bar{x})^2$
4	25
6	9
7	4
8	1
10	1
12	9

Question 34

Let $\bar{x}$ denote the mean of the observations 1, 3, 5, a, 9 and $\bar{y}$ denote the

mean of the observations 2, 4, b, 6, 8 where a, b > 0. If $\overline{x} = \overline{y}$, the value of 2(a − b) is

Options:

A. 2

B. 38

C. 8

D. -4

E. 4

Answer: E

Solution:

Solution:

$\overline{x} = \overline{y}$

$$\frac{1 + 3 + a + 9 + 5}{5} = \frac{2 + 4 + b + 6 + 8}{5}$$

$18 + a = 20 + b$

$a - b = 2$

$2(a - b) = 4$

Question 35

Consider two independent events E and F such that $P(E) = \frac{1}{4}$, $P(E \cup F) = \frac{2}{5}$ and P(F) = a. Then, the value of a is

Options:

A. $\frac{13}{20}$

B. $\frac{1}{20}$

C. $\frac{1}{4}$

D. $\frac{1}{5}$

E. $\frac{3}{5}$

Answer: D

Solution:

Solution:

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$\frac{2}{5} = \frac{1}{4} + a - \frac{a}{4}$

Since E & F are independent

$a = \frac{2}{5}\left(-\frac{1}{4}\right)^{4}$

$\equiv \frac{(8-5)4}{20}$

$\frac{4}{20} = -\frac{1}{5}$

Question 36

There are two cash counters A and B for placing orders in a college canteen. Let E A be the event that there is a queue at counter A and E B denotes the event that there is a queue at counter B. If P(E A) = 0.45P(E B) = 0.55 and P(E A ∩ E B) = 0.25. then the probability that there is no queue at both the counters is

Options:

A. 0.75 B. 0.15 C. 0.25 D. 0.20
(E) 1.75

Answer: C

Solution:

Solution:

$P(E_A^1 \cap E_B^1) = 1 - P(E_A \cup E_B)$
$= 1 - 0.75 = 0.25$

Question 37

Let S = {a, b, c} be the sample space with the associated probabilities satisfying P(a) = 2P(b) and P(b) = 2P(c). Then the value of P(a) is

Options:

A. $\frac{1}{5}$

B. $\frac{2}{7}$

C. $\frac{7}{16}$

D. $\frac{4}{7}$

E. –

Answer: E

Solution:

Solution:

$P(a) = k$

$P(b) = \frac{k}{2}$

$P(c) = \frac{k}{4}$

$P(a) + P(b) + P(c) = 1$

$\Rightarrow k + \frac{k}{2} + \frac{k}{4} = 1$

$4k + 2k + k = 4$

$7k = 4$

$k = \frac{4}{7}$

Question 38

A coin is tossed thrice. The probability of getting a head on the second toss given that a tail has occurred in at least two tosses is

Options:

A. $\frac{1}{2}$

B. $\frac{1}{16}$

C. $\frac{1}{8}$

D. $\frac{4}{1}$

E. $\frac{3}{1}$

Answer: D

Solution:

Solution:

Tail occurred in at least 2 tosses ⇒ { TTH, THT, HTT, TTT }

∴ Required probability $= \frac{1}{4}$

Question 39

Let X be a random variable following Binomial distribution; Bin(n, p), where n is the number of independent Bernoulli trials and p is the probability of success. If E (X) = 1 and Var(X) = $\frac{4}{5}$, then the values of n and p are

Options:

A. $n = 5,\ p = \frac{4}{5}$

B. $n = 1,\ p = \frac{1}{5}$

C. $n = 1,\ p = 1$

D. $n = 5,\ p = \frac{1}{5}$

E. $n = 1,\ p = \frac{4}{5}$

Answer: D

Solution:

Solution:

$E(x) = np = 1$

$Var(x) = npq = \frac{4}{5}$

$\frac{npq}{np} = \frac{4}{5} = q$

$\therefore p = \frac{1}{5},\ n = 5$

Question 40

A box contains 10 coupons, labelled as 1, 2, .. . 10. Three coupons are 1, X 2 and X 3 denote

drawn at random and without replacement. Let X

the numbers on the coupons. Then the probability that max{X 1, X 2, X 3} < 7 is

Options:

A. $\frac{^3C_1}{^{10}C_3}$

B. $\frac{^7C_3}{^{10}C_3}$

C. $\frac{^3C_3}{^{10}C_3}$

D. $\frac{^3C_1}{^{10}C_7}$

E. $\frac{^6C_3}{^{10}C_3}$

Answer: E

Solution:

Solution:

$\frac{^{6}C_{3}}{^{10}C_{3}}$ (should not get 7, 8, 9)

Question 41

An electric bulb manufacturing company manufactures three types of electric bulbs A, B and C. In a room containing these three types of electric bulbs, it is known that 6% of type A electric bulbs are defective, 4% of type B electric bulbs are defective and 2% of type C electric bulbs are defective. An electric bulb is selected at random from a lot containing 50 type A electric bulbs, 30 type B electric bulbs and 20 type C electric bulbs. The selected electric bulb is found to be defective. Then the probability that the selected electric bulb was type A is

Options:

A. $\frac{2}{23}$

B. $\frac{23}{500}$

C. $\frac{12}{23}$

D. $\frac{15}{23}$

E. $\frac{115}{6}$

Answer: D

Solution:

Solution:

Applying Bayes' theorem

$$P[E_1 / A] = \frac{P[E_1][A / E_1]}{\sum_{i=1}^{3} P[E_i][A / E_i]}$$

$$= \frac{\frac{50}{100} \cdot \frac{6}{100}}{\frac{50 \cdot 6}{100 \cdot 100} + \frac{30 \cdot 4}{100 \cdot 100} + \frac{20 \cdot 2}{100 \cdot 100}}$$

$$= \frac{15}{23}$$

Question 42

For four observations x_1, x_2, x_3, x_4, it is given that $\sum_{i=1}^{4} x_i^2 = 656$ and $\sum_{i=1}^{4} x_i = 32$. Then, the variance of these four observations is

Options:

A. 144

B. 730

C. 120

D. 248

E. 182.5

Answer: A

Solution:

Solution:

$$\text{variance} = \frac{\sum_{i=1}^{4} xi^2}{4} - (\bar{x})2$$

$$= \frac{656}{4} - \left(\frac{32}{4}\right)^2 = 164 - 64 = 100$$

Question 43

An um contains 8 black marbles and 4 white marbles. Two marbles are chosen at random and without replacement. Then the probability that both marbles are black is

Options:

A. $\frac{7}{33}$

B. $\frac{3}{2}$

C. $\frac{7}{11}$

D. $\frac{14}{33}$

E. $\frac{21}{143}$

Answer: D

Solution:

Solution:
Required probability

$$= \frac{8}{12}\cdot\frac{7}{11}\cdot\frac{14}{33}$$

Question 44

A box contains 100 tickets numbered 00, 01, 02, . . . 99 and a ticket is

drawn at random. Let X denote the sum of the digits on that ticket and Y denote the product of those digits. Then the value of P(X = 2 | Y = 0) is

Options:

A. $\frac{3}{19}$

B. $\frac{6}{19}$

C. $\frac{1}{19}$

D. $\frac{2}{19}$

E. $\frac{1}{100}$

Answer: D

Solution:

Solution:

P[x=2/y=0]

$= \frac{P(x=2 \cap y=0)}{P[y=0]}$

Favourable cases to y = 0

01, 02, 03, 04, 05, 06, 07, 08, 09, 10, 20,

30, 40, 50, 60, 70, 80, 90,

In this cases the tickets 02 and 20 are favourable to x = 2 and y=0

$\therefore$ Required probability $= \frac{2}{19}$

Question 45

Let the coefficient of variation of two datasets be 50 and 75. respectively and the corresponding variances be 25 and 36. respectively. Also let $\overline{x}_1$ and $\overline{x}2$ denote the corresponding sample means. Then $\overline{x}1 + \overline{x}2$ is

Options:

A. 2 B. 10

C. 18

D. 20

E. 16

Answer: C

Solution:

Solution:

$C \cdot V = \frac{SD}{\bar{x}_1} \cdot 100$

$= \frac{5}{\bar{x}_1} \cdot 100 = 50$

$\bar{x}_1 = \frac{500}{50} = 10$

$75 = \frac{6}{\bar{x}_2} \cdot 100$

$\Rightarrow \bar{x}_2 = \frac{600}{75} = 8$

$\bar{x}_1 + \bar{x}_2 = 18$

Question 46

The mean deviation about the median for the data 3, 5, 9, 3, 8, 10, 7 is

Options:

A. $\frac{23}{7}$

B. $\frac{4}{7}$

C. $-\frac{4}{7}$

D. $\frac{16}{7}$

E. $\frac{17}{7}$

Answer: D

Solution:

Solution:

$M \cdot D = \frac{\Sigma |d|}{n}$, n = 7

Median is middle most item in, 3, 3, 5, 7, 8 9 10

i.e., 7

$\therefore \Sigma_d = 4+4+2+0+1+2+3=16$

$\therefore M \cdot D = \frac{16}{7}$

Question 47

A biased die is rolled such that the probability of getting k dots, $1 \le k \le 6$. on the upper face of the dic is proportional to k. Then the probability that five dots appear on the upper face of the die is

Options:

A. $\frac{16}{21}$

B. $\frac{2}{21}$

C. $\frac{1}{21}$

D. $\frac{3}{5}$

E. $\frac{21}{}$

Answer: E

Solution:

Solution:

Sum of total probability = 1

$K + 2K + 3K + 4K + 5K + 6K = 1$

$\Rightarrow 21K = 1$

$K = \frac{1}{21}$

$\therefore 5K = \frac{5}{21}$

Question 48

Let Ω = {1, 2, 3, 4, 5} be the sample space with the events A = {1, 2, 5}, B={1,3,5} and C={2,3,}

5. Let E denote the complement of an event E. Then P((A∩B)c∪ cC) is

Options:

A. $\frac{1}{5}$

B. $\frac{3}{2}$

C. $\frac{5}{4}$

D. $\frac{5}{}$

E. 1

Answer: D

Solution:

Solution:

$A \cap B = \{1, 5\}$

$(A \cap B)^C = \{2, 3, 4\}$

$C^C = \{1, 4\}$

$(A \cap B)^C \cup C^C = \{1, 2, 3, 4\}$

required probability $= \frac{4}{5}$

Question 49

For any real number x, the least value of $4\cos x - 3\sin x + 5$ is

Options:

A. 10 B. 2 C. 0 D. 8 E. 4

Answer: C

Solution:

Solution:

Standard result

$c - \sqrt{a^2+b^2} \le a\cos\theta + b\sin\theta + c \le c + \sqrt{a^2+b^2}$

Here $c=5, \quad a=4, \quad b=3$

$\therefore$ Least value

$= c - \sqrt{a^2+b^2} = 5 - \sqrt{16+9} = 0$

Question 50

Let $P(x) = \quad \in \mathbb{R}$. Then which of the following options is correct for all x ?

Options:

A. $\frac{1}{6} \le P(x) \le \frac{3}{4}$

B. $0 \le P(x) \le \frac{1}{2}$

C. $0 \le P(x) \le 1$

D. $\frac{1}{2} \le P(x) \le \frac{3}{2}$

E. $\frac{3}{4} \le P(x) \le 1$

Answer: E

Solution:

Solution:

$p(x) = \cos^2 x + \quad ^2 (\quad - \cos^2 x)$

$= 1 - \sin^2 x \cos^2 x$

$= 1 - \frac{1}{4}\sin^2 2x$

$0 \le \sin^2 x \le 1$

$0 \ge -\frac{1}{4}\sin^2 x \ge \frac{-1}{4}$

$1 \ge 1 - \frac{1}{4}\sin^2 x \ge \frac{3}{4}$

i.e., $\frac{3}{4} \le p(x) \le 1$

Question 51

Let α and β be such that α + β = π. If $\cos\alpha = \frac{1}{\sqrt{2}}$, then the value of cot(β − α) is

Options:

A. ∞

B. 1

C. $\frac{1}{2}$

D. $\frac{1}{4}$

E. 0

Answer: D

Solution:

Solution:

$\cos\alpha = \frac{1}{\sqrt{2}} \Rightarrow \alpha = 45^\circ$

$\alpha + \beta = 180^\circ$

$\therefore \beta = 135^\circ$

$\cot(135 - 45) = \cot 90 = 0$

Question 52

The value of $\text{cosec}20^\circ \tan 60^\circ - \sec 20^\circ$ is

Options:

A. 0 B. 1 C. 2 D. 4 E. 6

Answer: D

Solution:

Solution:

$\text{cosec}20^\circ\tan 60^\circ - \sec 20^\circ$

$= \sqrt{3}\text{cosec}20 - \sin 20$

$\sqrt{\frac{\sqrt{3}}{\sin 20} - \frac{1}{\cos 20}}$

$\equiv \frac{\sqrt{3}\cos 20 - \sin 20}{\sin 20 \cos 20}$

$= \frac{\frac{\sqrt{3}}{2}\cos 20 - \frac{1}{2}\sin 20}{\frac{1}{2}\sin 20 \cos 20}$

$= \frac{\sin 60\cos 20 - \cos 60\sin 20}{\frac{1}{2}\frac{1}{2}\sin 40}$

$= \frac{\sin 40}{\frac{1}{4}\sin 40} = 4$

Question 53

If $\alpha + \beta + \gamma = 2\pi$, then the value of $\cot\frac{\alpha}{2}\cot\frac{\beta}{2} + \cot\frac{\alpha}{2}\cot\frac{Y}{2} + \cot\frac{\beta}{2}\cot\frac{Y}{2}$ is

Options:

A. 0 B. 1

C. $\frac{\pi}{2}$

D. $\frac{\pi}{3}$

E. $\frac{1}{2}$

Answer: B

Solution:

Solution:

$\alpha + \beta + \gamma = 2\pi$

conditional identity

$\cot\frac{\alpha}{2}\cot\frac{\beta}{2} + \cot\frac{\alpha}{2}\cot\frac{\gamma}{2}\cot\frac{\beta}{2}1\frac{\gamma}{2}$

Question 54

Let p, q and r be the real numbers such that $|r| > \sqrt{p^2 + q^2}$. Then the equation p cos θ + qsin θ = r has

Options:

A. exactly one real solution.

B. exactly two real solutions.

C. infinite number of real solutions.

D. no real solution

E. integer solutions.

Answer: D

Solution:

Solution:

$\sqrt{p^2+q^2}$ △ leftr |

pcosθ + qsinθ = r has real solution only

if r $< \sqrt{p^2+q^2}$

hence there is no real solution

Question 55

If $x \in (, 0\pi$ satisfies the equation $6^{1+\sin x+\sin^2 x+\ldots} = 36$, then the value of x is

Options:

A. 0

B. $\frac{\pi}{3}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{2}$

E. $\frac{\pi}{4}$

Answer: C

Solution:

Solution:

$6^{1+\sin x+\sin^2 x+\ldots\ldots} = 36$

$= 6^2$

i.e., $\frac{1}{1-\sin x} = 2$

$1 - 2 - 2\sin x$

$2\sin x = 1$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}$

Question 56

The value (s) of a(≠0) for which the equation $\frac{1}{2}(\quad)^2 \quad \sin\left(\frac{a}{x}\right)$ holds is/ are

Options:

A. $(4n + 1)\pi,\ n \in Z$

B. $2(n - 1)\pi,\ n \in Z$

C. $n\pi,\ n \in N$

D. $\frac{n\pi}{2},n\in N$

E. 1

Answer: A

Solution:

Solution:

$\frac{1}{2}(x-2)^2 + 1 = \sin\left(\frac{a}{x}\right)$

minimum value of $(x-2)^2 \geq 0$ and $\sin\left(\frac{a}{x}\right) \leq 1$

Therefore equality holds when LHS = 1 RHS $\Rightarrow x = 2$

$\sin\left(\frac{a}{2}\right) = 1 \Rightarrow \sin\frac{\pi}{2}$

$\therefore \frac{a}{2} = n\pi + (-1)^n \frac{\pi}{2}$

$a = 2n\pi + (-1)^n \pi$

When $n = 1, \theta = a = \pi$

$n = 1,\ a = 2\pi - \pi = \pi$

$n = 2, a = 4\pi + \pi = 5\pi$

$n = 3, a = 8\pi + \pi = 9\pi$

$\Rightarrow 4(n-1)\pi, n \in 2$

Question 57

If x is a real number such that tan x+ cotx = 2, then x =

Options:

A. $\left(n + \frac{1}{4}\right)\pi,n\in Z$

B. $(n + 1)\pi,\ n \in Z$

C. $n\left(\quad + \frac{1}{2}\right)\pi,n\in Z$

D. $n\pi,\ n \in Z$

E. $\frac{2}{3}n\pi,\ n \in Z$

Answer: A

Solution:

Solution:

$\tan x + \cot x = 2$

$\tan x + \frac{1}{\tan x} = 2$

$\tan^2 x + 1 = 2\tan x$

$\tan^2 x - 2\tan x + 1 = 0$

$(\tan x - 1)^2 = 0$

$\tan x = 1$

$\tan x = \tan \frac{\pi}{4}$

$x = n\pi + \frac{\pi}{4}$

$= \pi\left(n + \frac{1}{4}\right), n \in Z$

Question 58

If $\frac{1 + \sin x}{1 - \sin x} = \frac{(1 + \sin y)^3}{(1 - \sin y)^3}$ for some real values x and y, then $\frac{\sin x}{\sin y}$ =

Options:

A. $\frac{3 + 2\sin y}{+ 21\ 3\sin y}$

B. $\frac{+23\cos y}{+ 21\ 3\cos y}$

C. $\frac{+23\sin y}{- 21\ 3\sin y}$ $\frac{3 + 2\sin y}{- 21\ 3\cos y}$

D. $\frac{+ 21\ 3\sin y}{- 21\ 3\cos y}$

E. ———

Answer: A

Solution:

Solution:

$\frac{1 + \sin x}{1 - \sin x} = \frac{(1 + \sin y)^3}{(1 - \sin y)^3}$

$\frac{(1 + \sin x) + (1 - \sin x)}{(1 + \sin x) - (1 - \sin x)}$

$= \frac{(1 + \sin y)^3 + (1 - \sin y)^3}{(1 + \sin y)^3 - (1 - \sin y)^3}$

$\frac{2}{2\sin x} = \frac{2 + 6\sin^2 y}{6\sin y + 2\sin^3 y}$

$2[3\sin y + \sin^3 y] = 2\sin x[3 + 3\sin^2 y]$

$\sin y[3 + \sin^2 y] = \sin x[3 + 3\sin^2 y]$

$\frac{\sin x}{\sin y} = \frac{3 + \sin^2 y}{1 + 3\sin^2 y}$

Question 59

Let k be a real number such that $\sin \frac{3\pi}{14} \cos \frac{3\pi}{14} = k \cos \frac{\pi}{14}$. Then the value

of 4k is

Options:

A. 1 B. 2

C. 3 D. 4

E. 0

Answer: B

Solution:

Solution:

$\sin\frac{3\pi}{14}\cos\frac{3\pi}{14} = k\cos\frac{\pi}{14}$

$\frac{1}{2}\sin\frac{6\pi}{14} = k\cos\frac{\pi}{14}$

$\frac{1}{2}\cos\left[\frac{\pi}{2}-\frac{6\pi}{14}\right] = k\cos\frac{\pi}{14}$

$\frac{1}{2}\cos\left(\frac{7\pi}{14}\right) = k\cos\frac{\pi}{14}$

$\frac{1}{2}\cos\left(\frac{\pi}{2}\right) = k\cos\left(\frac{\pi}{14}\right)$

$k = \frac{1}{2}$

$4k = 2$

Question 60

In a triangle ABC, if $\cos^2 A - \sin^2 B + \cos^2 C = 0$, then the value of cos A cos B cos C is

Options:

A. $\frac{1}{4}$

B. 1

C. $\frac{\pi}{2}$

D. $\frac{1}{2}$

E. 0

Answer: E

Solution:

Solution:

A + B + C = π

$\cos^2 A - \sin^2 B + \cos^2 C = 0$
$\cos(A + B)\cos(A - B) + \cos^2 C = 0.$
$\cos(\pi - C)\cos(A - B) + \cos^2 C = 0$
$-\cos C \cos(A - B) + \cos^2 C = 0$
$\cos C[\cos C - \cos(A - B)] = 0$
$\cos C[-\cos(A + B) - \cos(A - B)] = 0$
$\Rightarrow -\cos C(2 \cos A \cos B) = 0$
$\Rightarrow \cos A \cos B \cos C = 0$

Question 61

The value of $\cos^{-1}\left(\cos\frac{7\pi}{4}\right)$ is

Options:

A. 0

B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{4}$

E. $\frac{\pi}{6}$

Answer: D

Solution:

Solution:

$\cos^{-1}\cos\left(\frac{7\pi}{4}\right) = \cos^{-1}\cos\left(2\pi - \frac{\pi}{4}\right)$

$= \cos^{-1}\cos\left(\frac{-\pi}{4}\right) = \cos^{-1}\cos\frac{\pi}{4} = \frac{\pi}{4}$

Question 62

The value of $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{2}{5}\right)$ is

Options:

A. $\tan^{-1}(5)$

B. $\tan^{-1}\left(\frac{1}{5}\right)$

C. $\tan^{-1}\left(\frac{2}{3}\right)$

D. $\tan^{-1}\left(\frac{8}{9}\right)$

E. $\tan^{-1}\left(\frac{9}{8}\right)$

Answer: E

Solution:

Solution:

$\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{2}{5}$

$=\tan^{-1}\frac{\frac{1}{2}+\frac{2}{5}}{1-\frac{1}{2}\cdot\frac{2}{5}}$

$=\tan^{-1}\frac{\frac{9}{10}}{\frac{8}{10}}=\tan^{-1}\frac{9}{8}$

Question 63

The value of $\tan^{-1}(\sqrt{3})-\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is

Options:

A. $\frac{2\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{2}{\pi}$

E. $\frac{\pi}{6}$

Answer: E

Solution:

Solution:

$\tan^{-1}\sqrt{3}-\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

$=\frac{\pi}{3}-\frac{\pi}{6}=\frac{\pi}{6}$

Question 64

Let $\vec{a}=\hat{i}-\hat{j}+2\hat{k}$. Then the vector in the direction of $\vec{a}$ with magnitude 5 units is

Options:

A. $5\hat{i} - 5\hat{j} + 10\hat{k}$

B. $-5\hat{i} - 5\hat{j} + 10\hat{k}$

C. $\frac{1}{\sqrt{16}}(5\hat{i} - 5\hat{j} + 10\hat{k})$

D. $\frac{1}{\sqrt{6}}(-5\hat{i} - 5\hat{j} + 10\hat{k})$

E. $\frac{1}{\sqrt{6}}(-10\hat{i} - 5\hat{j} + 5\hat{k})$

Answer: C

Solution:

Solution:
Unit vector in the direction of
$\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$
$= \frac{\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{1+1+4}} = \frac{\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{6}}$
Vector with magnitude 5 = $\frac{5\hat{i} - 5\hat{j} + 10\hat{k}}{\sqrt{6}}$

Question 65

Let $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ be two vectors. Then the unit vector in the direction of $\vec{a} - \vec{b}$ is

Options:

A. $\frac{1}{\sqrt{10}}(2\hat{j} - 3\hat{k})$

B. $\frac{1}{\sqrt{10}}(3\hat{j} - \hat{k})$

C. $(3\hat{j} - \hat{k})$

D. $\frac{1}{\sqrt{5}}(2\hat{j} - 3\hat{k})$

E. $\frac{-1}{\sqrt{5}}(2\hat{j} - 3\hat{k})$

Answer: B

Solution:

Solution:
$\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ $\quad \vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$

$\bar{a} - \bar{b} = 0\hat{i} + 3\hat{j} - \hat{k}$
unit vector in the direction of
$\bar{a} - \bar{b} = \frac{3\hat{i} - \hat{k}}{\sqrt{9+1}} = \frac{3\hat{i} - \hat{k}}{\sqrt{10}}$

Question 66

The direction cosines of vector $\vec{a} = -2\hat{i} + \hat{j} - \hat{k}$ are

Options:

A. $\left(\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

B. $\left(\frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\right)$

C. $\left(\frac{-2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\right)$

D. $\left(\frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

E. $\left(\frac{-2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

Answer: B

Solution:

Solution:

direction cosines of $\bar{a} = -2\hat{i} + \hat{j} - \hat{k}$ is
$\frac{-2}{\sqrt{4+1+1}}, \frac{1}{\sqrt{4+1+1}}, \frac{-1}{\sqrt{4+1+1}}$
$= \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}$

Question 67

The value of λ for which the vectors $\hat{i} + \hat{j} - \hat{k}$ and $\lambda\hat{i} + 3\hat{j} + \hat{k}$ are perpendicular is

Options:

A. -2 B. 2 C. 0 D. 1 E. -1

Answer: A

Solution:

Solution:

Since vector are perpendicular, $\bar{a} \cdot \bar{b} = 0$

$\Rightarrow \lambda + 3 - 1 = 0$

$\Rightarrow \lambda = -2$

Question 68

The position vectors of two points P and Q are given $\vec{OP} = 2\vec{a} - \vec{b}$ and $\vec{OQ} = \vec{a} + 3\vec{b}$, respectively. If a point R divides the line joining P and Q internally in the ratio 1 : 2, then the position vector of the point R is

Options:

A. $\frac{1}{3}(5\vec{a} - \vec{b})$

B. $\frac{1}{3}(5\vec{a} + \vec{b})$

C. $\frac{1}{3}(\vec{a} - 5\vec{b})$

D. $\frac{1}{3}(\vec{a})$

E. $\frac{1}{3}(\vec{a} + \vec{b})$

Answer: B

Solution:

Solution:

$\bar{R} = \frac{(\bar{a} + 3\bar{b}) + 2(2\bar{a} - \bar{b})}{3}$

$= \frac{5\bar{a} + \bar{b}}{3}$

Question 69

Let $\vec{a}$ and $\vec{b}$ be perpendicular vectors such that $|\vec{a}| = 10$ and $|\vec{b}| = 6$. Then the value of $|\vec{a} - \vec{b}|$ is

Options:

A. $\sqrt{110}$

B. $\sqrt{140}$

C. $\sqrt{98}$

D. $\sqrt{55}$

E. $\sqrt{70}$

Answer: B

Solution:

Solution:

$|a| = \sqrt{104}$ $|b| = 6$ $a \perp b \Rightarrow a \cdot b = 0$

$|a - b|^2 = |a|^2 + |b|^2$

$= 104 + 36 = 140$

$|a - b| = \sqrt{140}$

Question 70

Let x be a real number and $\vec{a}$ be any non-zero vector such that $|(4-x)\vec{a}| < |3\vec{a}|$. Then which of the following options is correct?

Options:

A. $0 < x \leq 6$

B. $0 < x \leq 7$

C. $1 < x < 7$

D. $1 < x \leq 7$

E. $0 < x \leq 6$

Answer: C

Solution:

Solution:

$|(4-x)\vec{a}| < |3\vec{a}|$

$\Rightarrow |4 - x| < |3| = 3$

$-3 < 4 - x < 3$

$-7 < -x < -1$

$\Rightarrow 7 > x > 1$

Question 71

The value of λ for which the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $-4\hat{i} + \lambda\hat{j} - 8\hat{k}$ are collinear is

Options:

A. 0

B. 1

C. 3 D. 6

E. 4

Answer: D

Solution:

Solution:
Since the vector are collinear

$$\frac{2}{-4} = \frac{-3}{\lambda} = \frac{4}{-8}$$

$$\Rightarrow \lambda = 6$$

Question 72

The projection of the vector $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}$ is

Options:

A. $\frac{3}{4}$

4

B. $\frac{3}{2}$

C. $\frac{3}{1}$

D. 3

E. 0

Answer: B

Solution:

Solution:

$$\overline{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$$

$$\overline{b} = \hat{i} + 2\hat{j} + 2\hat{k}$$

projection of $\overline{a}$ and $\overline{b} = \frac{a \cdot b}{|b|} = \frac{2 - 6 + 8}{\sqrt{1+4+4}}$

$$= \frac{4}{3}$$

Question 73

Let $f(x) = \begin{cases} -5 & , x \leq 0 \\ x - 5 & , x > 0 \end{cases}$ and $g(x) = |f(x)| + f(|x|)|$

Then g(−2) will be

Options:

A. -1

B. -15

C. 1

D. 0

E. -11

Answer: A

Solution:

Solution:

$g(-2) = |f(-2)| + 2f(|-2|)$
$= |-5| + 2(2 - 5)$
$= 5 - 6 = -1$

Question 74

Let [.] denote the greatest integer function and f(x)=[x]+|2−x|,−1≤x≤4 Then

Options:

A. f is continuous at x $= 2$

B. f is not continuous at x $= 1$

C. f is continuous at x $= 0$

D. f is differentiable at x $= 3$

E. f is not differentiable at x $= \frac{3}{2}$

Answer: B

Solution:

Solution:
Since greatest integer function is not continuous at 1 .

Question 75

$\lim_{x \to 0} \frac{e^x - 1}{3(1 - e^{2x})} =$

Options:

A. $\frac{1}{6}$

B. $-\frac{1}{6}$

C. 3

D. 0

E. $-\frac{1}{3}$

Answer: B

Solution:

Solution:
Applying LHospitals rule
$\lim_{x\to 0} \frac{e^x}{-6e^{2x}} = \frac{-1}{6}$

Question 76

Let $f(x) = \left(\frac{1}{x}\right)^2$, x > 0. Then

Options:

A. f is increasing in (0, 2) and decreasing in (2, ∞)

B. f is decreasing in (0, 2) and increasing in (2, ∞)

C. f is increasing in (0, 1) and decreasing in (1, ∞)

D. f is decreasing in (0, 1) and increasing in (1, ∞)

E. f is increasing in (0, ∞)

Answer: D

Solution:

Solution:
$f(x) = \left(\frac{1}{x}\right)^2$
$f'(x) = 2\left(\frac{1}{x}\right)\left(-\frac{1}{2x}\right) +$
$\Rightarrow +2\frac{(x-)}{x}$

+	t	
0		1

∴ f is decreasing in (0,1) and increasing in (1,∞)

Question 77

Let R → R defined by

$$f(x) = \begin{cases} 3e^x & \text{if } x<0 \\ x^2 \pm 3x + 3 & \text{if } 0 \le x<1 \\ x^2 - 3x - 3 & \text{if } x \ge 1 \end{cases}.$$

Options:

A. f is continuous on R B. f is not continuous on R

C. f is continuous on R\{0} D. f is continuous on R\{1}

E. f is not continuous on R\{0, 1}

Answer: D

Solution:

Solution:
at $x = 0$
LHL = RHL
At $x = 1$
LHL $= 1 \mp 3 \mp 3 \mp = $
RHL $= 1 \mp 3 \mp 3 - = -5$
$\therefore f$ is not continuous at $x = 1$
$\therefore f$ is continuous on $R\{1\}$

Question 78

Let $f(x)=\pi \cos x + x^2$. The value of $c \in (0, \pi)$ where f attains its local maximum / minimum is

Options:

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. $\frac{3\pi}{4}$

D. $\frac{\pi}{3}$

E. $\frac{\pi}{6}$

Answer: B

Solution:

Solution:

$f'(x) = -\pi \sin x + 2x$

At $\frac{\pi}{2}$, $f'(x) = 0$

$\Rightarrow$ local minimum at $x = \frac{\pi}{2}$

Question 79

The minimum of $f(x) = \sqrt{10 - x^2}$ in the interval $[-3, 2]$ is

Options:

A. $-\sqrt{4}$

B. $\sqrt{6}$

C. 1

D. 0

E. $\sqrt{10}$

Answer: C

Solution:

Solution:

$f(x) = \sqrt{10 - x^2}$

$f'(x) = \frac{1}{2\sqrt{10 - x^2}} - 2x = 0$

$\Rightarrow x = 0$

$f(-3) = \sqrt{10 - 9} = 1$

$f(2) = \sqrt{10 - 4} = \sqrt{6}$

$f(0) = \sqrt{10 - 0} = \sqrt{10}$

Question 80

The equation of the line passing through origin which is parallel to the tangent of the curve $y = \frac{x-2}{x-3}$ at $x = 4$ is

Options:

A. $y = 2x$

B. $y = -2x + 1$

C. $y = -x$

D. $y = x + 2$

E. $y = 4x$

Answer: C

Solution:

Solution:

$y = \frac{x-2}{x-3}$

$\frac{dy}{dx}$ at $x = 4, \frac{dy}{dx} = \frac{(x-3)-(x-2)\times}{(x-3)^2} = \frac{1-}{1^2} = -1$

$\therefore$ required lime $\Rightarrow y - 0 = -(1 \times 0)$

$\Rightarrow y = -x$

Question 81

Let $f(x) = \alpha \sin^2 3x$. If $f'\left(\frac{\pi}{12}\right) = 3$, then the value of α is

Options:

A. -1

B. $-\pi$

C. π

D. $\frac{\pi}{2}$

E. 1

Answer: A

Solution:

Solution:

$f'(x) = \alpha 6 \sin 3x \cos 3x$

$f'\left(\frac{\pi}{12}\right) = \alpha 6 \sin \frac{3\pi}{12} \cos \frac{3\pi}{12} = -3$

$\Rightarrow 6 \times \alpha \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = -3$

$\Rightarrow 3\alpha = -3 \Rightarrow \alpha = -1$

Question 82

Let f : R → R be defined by
$ f(x)={\{\table 2 x+3, , x ≤ 5 ; 3 x+α, , x>5 }.
$
Then the value of α so that f is continuous on R is

Options:

A. 2

B. -2

C. 3

D. -3

E. 0

Answer: B

Solution:

Solution:
Since f is continuous, $10+3=15+\alpha$
$\alpha = -2$

Question 83

If $y = x^{e^x} + x^e$ for $x > 0$, then $\frac{dy}{dx}$ is equal to

Options:

A. $x^{e^x}\left[\frac{1}{x} + \ln x\right] + xe$

B. $x^{e^x} \cdot x\left[\frac{1}{x} + \ln x\right] + ex^{e-1}$

C. $e^x \cdot x^{e^{x-1}} + eex$

D. $x^{e^x} e^{-x}\left[\frac{1}{x} - \ln x\right] + ex^{e-1}$

E. $x^{e^x} \cdot x\left[\frac{1}{n} - \ln x\right] + ex^{e-1}$

Answer: B

Solution:

Solution:
$y = x^{e^x} + x^e$
$= u + v$
$u = x^{e^x}$
$\log u = e^x \log x \quad \frac{v = x^e}{dv} = ex^{e-1}$
$\frac{1}{u}\frac{du}{dx} = \frac{e}{x} + \log u \, e^x \times x$
$\frac{du}{dx} = ue^x\left(\frac{e}{x} + \log u \times e\right)$
$\therefore \frac{dy}{dx} = xe^{x^x}\left[\frac{1}{x} \log x\right] + ex^{e-1}$

Question 84

$\lim_{x \to 0} \frac{\ln(1 + (\ln 5)x)}{5^x - 1}$

Options:

A. 1

B. ln 5

C. -1

D. 5

E. $\frac{1}{5}$

Answer: A

Solution:

Solution:
Applying LH ospitals rule

$\lim_{x \to 0} \frac{\frac{1}{1 + (\ln 5)x} \times \ln 5}{5^x \ln 5} = 1$

Question 85

$\int \frac{1}{x^2 - 2x + 2} \mathbf{dx} =$

Options:

A. $\tan^{-1}(x-1)+C$

B. $\sin^{-1}(2x-1)+C$

C. $\sin^{-1}(x-1)+C$

D. $\tan^{-1}(2x-1)+C$

E. $\frac{1}{(2x-1)^3} + C$

Answer: A

Solution:

Solution:

$\int \frac{1}{x^2 - 2x + 2} dx = \int \frac{1}{(x-1)^2 + 1} dx$

$= \tan^{-1}(x - 1) + c$

Question 86

$\int \sin^2 \pi x dx =$

Options:

A. $\frac{\pi}{2} - \frac{1}{4\pi}\sin 2\pi x + C$

B. $\frac{2}{\pi} - \frac{1}{8\pi}\sin 4\pi x + C$

C. $\frac{8}{x} - \frac{1}{4\pi}\sin 2\pi x + C$

D. $x + \frac{1}{2\pi}\sin 2\pi x + c$

E. $\frac{x}{2} - \frac{2\pi}{1}\cos 2\pi x + C$

Answer: A

Solution:

Solution:

$\int \sin^2 \pi x dx = \int \frac{1-\cos 2\pi x}{2} dx$

$= \frac{x}{2} - \frac{1}{2}\frac{\sin 2\pi x}{2\pi} + c$

$= \frac{x}{2} - \frac{1}{4\pi}\sin 2\pi x + c$

Question 87

$\int \frac{x+5}{x^2-1} dx =$

Options:

A. $3\ln|x-1| - 2\ln|x+1| + C$

B. $2\ln|x-1| - 3\ln|x+1| + C$

C. $\ln|x-2| \ln|x+1| + C$

D. $\ln|x+2| - \ln|x-1| + C$

E. $2\ln|x-1| + 3\ln|x+1| + C$

Answer: A

Solution:

Solution:

$\int \frac{x+5}{x^2-1} dx = \int \frac{x+5}{(x-1)(x+1)} dx$

$\frac{x+5}{(x-1)(x+1)} = \frac{A}{x+1} + \frac{B}{x-1}$

$x + 5 = A(x-1) + B(x+1)$

$6 = 2B \quad 4 = -2A$

$B = 3 \quad A = -2$

$\therefore \int \left(\frac{-2}{x+1} + \frac{3}{x-1}\right) dx$

$= -2\ln|x+1| + 3\ln|x-1| + c$

Question 88

$\int \frac{2\tan x + 3}{\sin^2 x + 2\cos^2 x} dx =$

Options:

A. $\frac{3}{\sqrt{2}} \sin^{-1}\left(\frac{\sin x}{\sqrt{2}}\right) + \ln\left|\sin^2 x + 2\right| + C$

B. $\frac{3}{\sqrt{2}} \tan^{-1}\left(\frac{\tan x}{\sqrt{2}}\right) + \ln\left|\tan^2 x + 2\right| + C$

C. $\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{\tan x}{\sqrt{2}}\right) - \ln\left|\tan^2 x + 2\right| + C$

D. $\frac{3}{\sqrt{2}} \cos^{-1}\left(\frac{\cos x}{\sqrt{2}}\right) + \ln\left|\sin^2 x + 2\right| + C$

E. $\frac{1}{\sqrt{2}} \cos^{-1}\left(\frac{\cos x}{\sqrt{2}}\right) - \ln\left|\cos x^2 + 2\right| + C$

Answer: B

Solution:

Solution:

$\int \frac{2\tan x + 3}{\sin^2 x + 2\cos^2 x}$

$\div$ by $\cos^2 x \Rightarrow \int \frac{(2\tan x + 3)\sec^2 x dx}{\tan^2 x + 2}$

$u = \tan^2 x$

$du = \sec^2 x dx$

$\Rightarrow \int \left(\frac{2u+3}{u^2+2}\right) du = 2\int \frac{u du}{u^2+2} + 3\int \frac{du}{u^2+2}$

$= \ln\left|\tan^2 x + 2\right| + \frac{3}{\sqrt{2}} \tan^{-1}\left(\frac{\tan x}{\sqrt{2}}\right) + C$

Question 89

$\int x \log(1 + x^2) dx =$

Options:

A. $\frac{1}{2}(1 + x^2)\log(1 + x^2) + \frac{x^2}{2} + C$

B. $\frac{1}{2}(1 + x^2)\log(1 + x^2) - \frac{x^2}{2} + C$

C. $\frac{1}{2}(1 + x^2)\log(2 + x^2) - \frac{x^2}{2} + C$

D. $(1 + x^2)\log(1 + x^2) + (1 + x^2)$ C

E. $(1 - x^2)\log(1 + x^2) + (1 - x^2 +$ C

Answer: B

)

Solution:

+

Solution:

$\int x\log(1 + 2x)\,dx$

$= \log(1 + 2x)\cdot \frac{x^2}{2} - \int \frac{2x}{(1 + 2x)}\cdot \frac{x^2}{2}\,dx$

$= \frac{x^2}{2}\log(1 + 2x) - \frac{1}{2}\int \frac{(u - 1)\,du}{u}$

where $u = 2x + 1$

$= \frac{x^2}{2}\log(1 + 2x) - \frac{1}{2}(2x + 1) + \frac{1}{2}\log(2x + 1) + C$

$= \frac{1}{2}(1 + 2x)\log(1 + 2x) - \frac{x^2}{2} + C$

Question 90

Let R → R be defined by $f(x) = \begin{cases} x & \text{if } x \le 1 \\ -x + 2 & \text{if } x > 1 \end{cases}$. Then $\int_0^2 f(x)\,dx =$

Options:

A. $\frac{\pi}{2}$

B. 1

C. 2

D. 4

E. $\frac{\pi}{6}$

Answer: B

Solution:

Solution:

$f(x) = x$ if $x \le 1$

$-x + 2$ if $x > 1$

$\int_0^2 f(x)\,dx = \int_0^1 x\,dx + \int_1^2 (-x + 2)\,dx$

$= \left(\frac{x^2}{2}\right)_0^1 + \left(\frac{-x^2}{2} + 2x\right)_1^2$

$= \frac{1}{2} - 0 - 2 + 4 + \frac{1}{2} - 2 = 1$

Question 91

$$\int \frac{1}{\cos x(\sin x + 2\cos x)}\,dx =$$

Options:

A. $\ln|1 - \tan x| + C$

B. $\ln|3 + \sin x| + C$

C. $\ln|2 + \tan x| + C$

D. $\ln|1 + 2\sec x| + C$

E. $\ln|2 - \tan x| + C$

Answer: C

Solution:

Solution:

$$\int \frac{dx}{\cos x(\sin x + 2\cos x)}$$

$$\div \text{ by } \cos x \int \frac{dx}{\cos^2 x(\tan x + 2)}$$

$$= \int \frac{\sec^2 x dx}{\tan x + 2}$$

$$= \ell n|2 + \tan x| + c$$

Question 92

$$\int_0^2 \frac{2e^x}{1 + e^{2x}}\,dx =$$

Options:

A. $4(\tan^{-1}2 - \pi)$

B. $2\left(\tan e^{-1} - \frac{\pi}{2}\right)$

C. $2\left(\tan^{-1}e + \frac{\pi}{4}\right)$

D. $2\left(\tan e^{-1} - \frac{\pi}{4}\right)$

E. $4(\tan^{-1}2 + \pi)$

Answer: D

Solution:

Solution:

$$\int_0^1 \frac{2e^x}{1 + e^{2x}}du$$

put $u = e^x$

when $x = 0 \Rightarrow u = 1$ =

$du = e^x dx$ $= 1u$ $= e$

$\Rightarrow \int_1^e \frac{2du}{1+u^2} = (2\tan^{-1}u)_1^e = 2\left(\tan^{-1}e - \frac{\pi}{4}\right)$

Question 93

$$\int_0^2 (5xe^{2x} - \tan\frac{\pi}{4})dx$$

Options:

A. $\frac{5}{4}e^2 + \frac{1}{4}$

B. $-\frac{5}{4}2e - \frac{1}{4}$

C. $\frac{5}{4}e^2 - \frac{9}{4}$

D. $\frac{3}{4}e^2 + \frac{1}{4}$

E. $\frac{1}{4}e^2 + \frac{5}{4}$

Answer: A

Solution:

Solution:

$\int_0^1 (5xe^{2x} - \tan\frac{\pi}{4})dx$

$= \left(5x\frac{e^{2x}}{2}\right)_0^1 - \int_0^1 5\frac{e^{2x}}{2}dx - \int_0^1 \tan\frac{\pi}{4}dx$

$= \frac{5}{2}e - \frac{5}{4}(e^{2x})_0^1 - \tan\frac{\pi}{4}(1)_0^1$

$= \frac{5}{2}e^2 - \frac{5}{4}e^2 + \frac{5}{4} - 1 = \frac{5}{4}e^2 + \frac{1}{4}$

Question 94

The area of the region in the first quadrant enclosed by the curves $y = \sqrt{x}$, $y = -x + 6$ and the x-axis is

Options:

A. $\frac{22}{7}$

B. $\frac{22}{3}$

C. 12

D. 24

E. 8

Answer: B

Solution:

Solution:

Solving $-x+=6\sqrt{}$, we get $=\ x\quad 4$

$\int_0^4 \sqrt{x}dx + \int_4^6 (6-x)dx$

$= \left(\frac{3}{2x\overline{2}}\right)_0 + \left(6x - \frac{x^2}{2}\right)_4^6 = \frac{22}{3}$

Question 95

The area of the region in the first quadrant which is above the parabola $y = 2x$ and enclosed by the circle $x^2+2 y=2$ and the y-axis is

Options:

A. $\frac{1}{6} + \frac{\pi}{4}$

B. $\frac{1}{12} + \frac{\pi}{6}$

C. $-\frac{1}{6} + \frac{\pi}{4}$

D. $\frac{1}{4} + \frac{\pi}{6}$

E. $-\frac{\pi^2}{2} + 4$

Answer: A

Solution:

Solution:

$\therefore$ required area $\int_0^1 \sqrt{2-2x}\, dx - \int_0^1 x^2 dx = \frac{\pi}{4} + \frac{1}{6}$

Question 96

$\int_0^2 \frac{x}{x2-4}$ **dx =**

Options:

A. $-\frac{\pi^2}{6}$

B. $-\frac{22}{7}$

C. $\ln\left(\frac{\sqrt{3}}{2}\right)$

D. $\ln\left(\frac{3}{2}\right)$

E. $\ln\left(\frac{3}{\sqrt{2}}\right)$

Answer: C

Solution:

Solution:

$$\int_0^1 \frac{x}{x^2-4}\,dx$$

$u = x^2 - 4$

$du = 2x\,dx$

$\Rightarrow \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\log u$

$= \left[\frac{1}{2}\log(x^2-4)\right]_0^1$

$= \frac{1}{2}[\log(3) - \log(-4)]$

$\equiv \frac{1}{2}\left[\log\left(\frac{-3}{4}\right)\right]$

$\log\frac{\sqrt{3}}{2}$

Question 97

If (2, −6), (5, 2) and (−2, 2) constitute the vertices of a triangle, then the line joining orgin and its orthocentre is

Options:

A. x + 4y = 0

B. x − 4y = 0

C. 4x − y = 0

D. 4x + y = 0

E. x−y=0

Answer: B

Solution:

Solution:

Slope of AB = 0

Slope of CD = $\frac{1}{0}$

Eq of CD ⇒ $y + 6 = \frac{1}{0}(x - 2)$

⇒x = 2 ····· · (i)

Slope of BC= $\frac{8}{-4} = -2$

Slope of AE = $\frac{1}{2}$

Eq of AE $\Rightarrow y - 2 = \frac{1}{2}(x - 5)$

$x - 2y = 1$

sub (1) in (2)

$2 - 2y = +1$

$-2y = -1$

$y = \frac{1}{2}$

Orthocentre

Eq of line joining $(0, 0)$ and $(2, \frac{1}{2})$ is

$y - 0 = \frac{1}{4}(x - 0)$

$\Rightarrow x - 4y = 0$

Question 98

If a straight line in X Y plane passes through

$(-a, -b), (a, b), (k, k)$ and $(2, 3)$, where $ab \neq 0$, then which of the following options is correct?

Options:

A. $k = 0$ when $a \neq b$

B. k is necessarily a positive real number when a

C. k is any positive real number when $a \neq b$ $= b$

D. $k = a$ or $k = b$ necessarily

E. $k \neq 0$ when $a \neq b$

Answer: A

Solution:

Solution:

Since points are co-planar

$$\begin{vmatrix} -a & -b & 1 \\ a & b & 1 \\ k & k & 1 \end{vmatrix} = 0$$

$R1 \rightarrow R1 + R2 \Rightarrow \begin{vmatrix} 0 & 0 & 2 \\ a & b & 1 \\ k & k & 1 \end{vmatrix} = 0$

$2(ak - bk) = 0$

$\Rightarrow a - b = 0$ or

$k = 0$

Question 99

The line perpendicular to $4x - 5y + 1 = 0$ and passing through the point of intersection of the straight lines $x + 2y - 10 = 0$ and $2x + y + 5 = 0$ is

Options:

A. $5x + 4y = 0$

B. $y + \frac{5}{4}x = \frac{50}{3}$

C. $5x + 4y = 1$

D. $y + \frac{5}{4}x = -\frac{50}{3}$

E. $4x + 5y = 0$

Answer: A

Solution:

Solution:

$(x + 2y - 10) + \lambda(2x + y + 5) = 0$

$\Rightarrow (2\lambda + 1)x + (2 + \lambda)y + (-10 + 5\lambda) = 0 - - (1)$

$\frac{-(2\lambda + 1)}{2 + \lambda} \times \frac{4}{5} = -1$

$8\lambda + 4 = 10 + 5\lambda$

$3\lambda = 6$

$\Rightarrow \lambda = 2$

Sub in (1) $5x + 4y = 0$

Question 100

A thin particle move from (0, 1) and gets reflected upon hitting the x-axis at ($\sqrt{3}$, 0). Then the slope of the reflected line is

Options:

A. $\frac{1}{\sqrt{3}}$

B. $-\frac{1}{\sqrt{3}}$

C. $\sqrt{3}$

D. $-\sqrt{3}$

E. 0

Answer: A

Solution:

Solution:

$m = \frac{0 - 1}{\sqrt{3} - 0} = \frac{-1}{\sqrt{3}}$

$\therefore$ slope of reflected line $= \frac{1}{\sqrt{3}}$

Question 101

If the two sides AB and AC of a triangle are along 4x − 3y − 17 = 0 and 3x + 4y − 19 = 0, then the equation of the bisector of the angle between AB and AC is

Options:

A. x − 7y + 2 = 0

B. 7x + y − 36 = 0

C. 7x − y + 36 = 0

D. x = y

E. x − 7y + 2 = 0

Answer: E

Solution:

Solution:
$4x-3y-17=0$
$3x+4y-19=0$
Solving, we get $x=5, y=1$ $x-7y+2=0$ satisfies this point

Question 102

A point moves in such a way that it remains equidistant from each of the lines 3x ± 2y = 5. Then the path along which the point moves is

Options:

A. $x = -\frac{5}{3}$

B. $y = -\frac{5}{3}$

C. $x = \frac{5}{3}$

D. $y = -\frac{5}{3}$

E. x = 0

Answer: C

Solution:

Solution:
$3x+2y=5$ $3x-2y=5$
Solving, we get $x = \frac{5}{3}$. this is the bisector

Question 103

Suppose the line mx − y + 5m − 4 = 0 meets the lines x + 3y = 2 = 0, 2x + 3y + 4 = 0 and x − y − 5 = 0 at the points R, S and T , respectively. If R, S and T at distances r1, r2 and r3 respectively, from (−5, −4) and $\frac{15}{r_1}+\frac{10}{r_2}=\frac{6}{r_3}$ then the value of m is

Options:

A. $-\frac{2}{3}$

B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. $-\frac{3}{2}$

E. 18

Answer: E

Solution:

Solution:

Coordinates of R will be

$(-5+r_1\cos\theta, -4+r_1\sin\theta)$

Sub in $x+3y+2=0$

$-5+r_1\cos\theta+3(-4+r_1\sin\theta)+2=0$

$r_1(\cos\theta+3\sin\theta)=15$

$r_1=\frac{15}{\cos\theta+3\sin\theta}$

Coordinates of S will be

$(-5+r_2\cos\theta, -4+r_2\sin\theta)$

Sub in $2x+3y+4=0$

$2(-5+r_2\cos\theta)+3(-4+r_2\sin\theta)+4=0$

$r_2(2\cos\theta+3\sin\theta)=18$

$r_2=\frac{18}{2\cos\theta+3\sin\theta}$

Coordinates of T will be

$(-5+r_3\cos\theta, -4+r_3\sin\theta)$

$\Rightarrow r_3=\frac{6}{\cos\theta+\sin\theta}$

Substituting in

$\left(\frac{15}{r_1}\right)+\left(\frac{10}{r_2}\right)=\left(\frac{6}{r_3}\right)$

we get $\tan\theta=18$

now slope of $mx-y+5m-4=0$ is m

$\therefore m=\tan\theta=18$

Question 104

Suppose the point P(1, 1) is translated to Q in the direction of y = 2x. If

PQ = 1, then Q is

Options:

A. (2, 0)

B. (0, 2)

C. $\left(\frac{\sqrt{2}+1}{\sqrt{2}}, \frac{\sqrt{2}+1}{\sqrt{2}}\right)$

D. $\left(\frac{\sqrt{5}+1}{\sqrt{5}}, \frac{\sqrt{5}+2}{\sqrt{5}}\right)$

E. $\left(\frac{2+\sqrt{3}}{2}, \frac{3}{2}\right)$

Answer: D

Solution:

Solution:

point (1, 1) PQ = 1 = r

Translation x 1 + rcosθ, y 1 + rsin θ

$\left(1+\frac{1}{\sqrt{5}}, 1+\frac{2}{\sqrt{5}}\right)$

$= \left(\frac{\sqrt{5}+1}{\sqrt{5}}, \frac{\sqrt{5}+2}{\sqrt{5}}\right)$

Question 105

Suppose the line joining distinct points P and Q on $(x-)^2+(-)^2=2$ 2y1r is the diameter of x1y $(-\quad)+(-\quad)^2 3 = 4$. Then the value of r is

Options:

A. 2 B. 3 C. 1

D. 9 E. 4

Answer: B

Solution:

Solution:

S_1 : $(x-2)^2+(y-1)^2 \equiv r^2$

S_1 : $(x-1)^2+(y-3)^2 4$

$S_1-S2=0$

$-2x + 4y - 5 = 2r-4$

Now (1, 3) lie on this
$-2+\ 12-5=r^2-4$
$r^2=9$
$\Rightarrow r=3$

Question 106

The equation of the circle that can be inscribed in the square formed by

$x^2-8\ \ x+12=0$ and $y^2-14y+45=0$ is

Options:

2

A. $x\ -8x-14y+61=0$

B. $x^2-8x-14y+71=0$

C. $x^2-4x-7y+61=0$

D. $x^2-4x-7y+71=0$

E. $x^2+y^2-8x-14y+61=0$

Answer: E

Solution:

Solution:

$x^2-8x+12=0\ y\ =9,5$
$x=6,2\ y\ =14y+45=0$
$\Rightarrow$ centre $=(,4\ 7\ )$
Radius $=2$
$\Rightarrow(x-\ 4)^2+(-y\ \ 7)^2=\ 4$
$\Rightarrow x^2+y^2-8x-14y+61=0$

Question 107

For the circle C $x^2+y^2-6x+2y=0$, which of the following is incorrect

Options:

A. the radius of C is $\sqrt{10}$

B. (3, −1) lies inside of C

C. (7, 3) lies outside of C

D. the line x + 3y = 0 intersects C

E. one of diameter of C is not along x + 3y = 0

Answer: E

Solution:

Solution:

$x^2 + y^2 - 6x + 2y = 0$

$g = -3,\ f = 1$

centre $= (3, -1)$ $r = \sqrt{9 + 1} = \sqrt{10}$

substituting $(3, -2)$, $1 + 9 + 1 - 18 - 2 < 0$

substituting $(7, 3)$, $49 + 9 - 42 + 6 > 0$

$(3, -1)$ satisfy $x + 3y = 0$

Question 108

For i = 1, 2, 3, 4, suppose the points $(\cos\theta_i, \sec\theta_i)$ lie on the boundary of a circle, where $\theta_i \in [0, \frac{\pi}{2})$. If $\theta_1, \theta_2, \theta_3, \theta_4$ are distinct. Then $\cos\theta_1 \cos\theta_2 \cos\theta_3 \cos\theta_4$ equals

Options:

A. $\frac{1}{2}$

B. $\frac{1}{4}$

C. $\frac{1}{8}$

D. $\frac{1}{16}$

E. 1

Answer: E

Solution:

Solution:

points are $(\cos\theta, \sec\theta) = (\cos\theta, \frac{1}{\cos\theta})$

$x^2 + 1 = 2xr$

$\Rightarrow x^4 + 1 = 2x^2$

$\Rightarrow x_1x_2x_3x_4 = 1$

(using relations between coefficient and roots)

Since all point lie on the circle,

$\cos\theta_1 \cos\theta_2 \cos\theta_3 \cos\theta_4 = 1$

Question 109

The set of points of the form $(t^2 + t + 1, t^{2+} - t + 1)$ where t is a real number, represents a / an

Options:

A. circle

B. parabola

C. ellipse

D. hyperbola

E. pair of straight line

Answer: B

Solution:

Solution:

$x = t^2 + t + 1 \quad y = t^2 - t + 1$

$x + y = 2t^2 + 2 - (1)$

$x - y = 2t$

$t = \frac{x - y}{2}$

Sub in (1)

$x + y = \frac{2(x-2y}{4} + 2,$ represents a parabola

Question 110

Suppose a and b are the lengths of major and minor axes of an ellipse that passes through the points (4, 3) and (−1, 4). If the major axis of the ellipse lies along the x-axis, then the value of $\frac{1}{a^2} + \frac{16}{b^2}$ is

Options:

A. 4

B. $\frac{1}{4}$

C. 2

D. $\frac{1}{2}$

E. 1

Answer: A

Solution:

Solution:

Let the equation of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Since it passes through $(-1,4)$ $\frac{1}{a^2} + \frac{16}{b^2} = 1$

Question 111

For a real number t, the equation $(1 + t)x^2 + (t - 1)y^2 + t^2 - 1 = 0$ represents a hyperbola provided

Options:

| | < 1 | | > 1 | | = 1

A. t

B. t

C. t

D. t ∈ (1, ∞]

E. t ∈ (−∞, −1]

Answer: A

Solution:

Solution:

$(1 + t)x^2 + (t - 1)y^2 + t^2 - 1 = 0$

$\frac{x^2}{1 - t} - \frac{y^2}{1 + t} = 1$

$1 - t > 0$

$\Rightarrow t < 1$

$\Rightarrow |t| < 1$

Question 112

Given the points A(6, −7, 0), B(16, −19, −4). C(0, 3, −6) and D(2, −5, 10), the point of intersection of the lines AB and CD is

Options:

A. (−1, 1, 2) B. (1, −1, 2)

C. (1, −1, −2) D. (−1, 1, −2)

E. (1, 1, 2)

Answer: B

Solution:

Solution:

AB: $\frac{x - 6}{10} = \frac{y + 7}{-12} = \frac{z - 0}{-4} \equiv \lambda$

CD : $\frac{x - 0}{2} = \frac{y - 3}{-8} = \frac{z + 6}{16}$

General point AB $\equiv (10\lambda + 6, -12\lambda - 7, -4\lambda)$

General point CD $\equiv (2\mu, -8\mu + 3, 16\mu - 6)$

Solving $\mu = \frac{1}{2}$

$\Rightarrow$ required point $= (1, -1, 2)$

Question 113

If the xz-plane divides the straight line joining the points (2, 4, 7) and (3, −5, 8) in the ratio α : 1, then the value of α is

Options:

A. $\frac{5}{4}$

B. $\frac{1}{3}$

C. $\frac{7}{4}$

D. $\frac{8}{5}$

E. $\frac{5}{2}$

Answer: D

Solution:

Solution:

Since xz plane, $\Rightarrow y = 0$

$\Rightarrow \frac{-5\alpha + 4}{\alpha + 1} = 0$

$\Rightarrow \alpha = \frac{4}{5}$

Question 114

If θ_1, θ_2 and θ_3 are the angles made by a line with the positive directions of the x, y, z axes, then the value of $\cos 2\theta_1 + \cos 2\theta_2 + \cos 2\theta_3$ is

Options:

A. -1 B. 1 C. 2 D. -2 E. 0

Answer: A

Solution:

Solution:

$\cos^2\theta_1 + \cos^2\theta_2 + \cos^2\theta_3 = 1$

$\cos 2\theta_1 + \cos 2\theta_2 + \cos 2\theta_3$
$= 2\cos^2\theta_1 - 1 + 2\cos^2\theta_2 - 1 + 2\cos^2\theta_3 - 1$
$= 2(1) - 3 = -1$

Question 115

The angle between the lines, whose direction cosines are proportional to 4, $\sqrt{3} - 1$, $-\sqrt{3} - 1$ and 4, $\sqrt{3} - 1$, is

Options:

A. $\frac{\pi}{6}$

B. $\frac{4}{\pi}$

C. $\frac{3}{\pi}$

D. $\frac{2}{\pi}$

E. π

Answer: A

Solution:

Solution:

$$\cos\theta = \frac{4 \times 4 + (\sqrt{3} - 1)(-1 - \sqrt{3}) + 1}{\sqrt{16 + (\sqrt{3} - 1)^2}\sqrt{1 + (-1 - \sqrt{3})^2}}$$

$$= \frac{12}{24} = \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{6}$$

Question 116

Suppose P is the point on the line joining $(-9, 4, 5)$ and $(11, 0, -1)$ that lies closest to the origin O. Then $|OP|^2$ equals to

Options:

A. 3 B. 4 C. 2

D. 9 E. 1

Answer: D

Solution:

Solution:

Given line is $\frac{x+9}{20} = \frac{y-4}{-4} = \frac{z-5}{-6}$

$|OP|^2$

$\frac{x-0}{20} = \frac{y-0}{-4} = \frac{z-0}{-6}$

$\therefore SD = \left|\left(\frac{9i+4j+5k}{\sqrt{400+16+36}}\right) \times (20i - 4j - 6k)\right| = 9$

Question 117

The plane that is perpendicular to the planes x 2x − 2y + z = 0 and passes through (1, −2, 1) is− y + 2z − 4 = 0 and

Options:

A. x y 1 + = 0

B. 2x y z 1 − = 0

C. x y z + = 0

D. 2x y z 1 + = 0

E. x z 2 − = 0

Answer: A

Solution:

Solution:

$\hat{n} = \frac{1}{\sqrt{}} \begin{vmatrix} i & j & k \\ 1 & -1 & 2 \\ 2 & 2 & 1 \end{vmatrix} = \frac{3i + 3j + 0k}{\sqrt{1}}$

$a(x -) + b(y + 2) + (z -) = 0$

$3(x - 1) + 3(y + 2) + 0 = 0$

$3x + 3y + 3 = 0$

$\Rightarrow x + y + 1 = 0$

Question 118

The line of intersection of the planes 3x 2x+y−2x−5=0 is − 6y − 2z − 15 = 0 and

Options:

A. $\frac{x+3}{1\,4} = \frac{y+1}{-2} = \frac{z}{15}$

B. $\frac{x+3}{-14} = \frac{y+1}{2} = \frac{z}{15}$

C. $\frac{x+3}{14} = \frac{y+1}{2} = \frac{z}{-15}$

D. $\frac{x+3}{14} = \frac{y-1}{-2} = \frac{z+1}{-15}$

E. $\frac{x-3}{14} = \frac{y+1}{2} = \frac{z}{15}$

Answer: E

Solution:

Solution:

$\bar{b} = \begin{vmatrix} i & j & k \\ 3 & -6 & -2 \\ 2 & 1 & -2 \end{vmatrix} = 14i+2j+15k$

Now $(3, -1, 0)$ satisfy
$3x - 6y - 2z - 15 = 0$

Question 119

The plane passing through the points (2, 1, 0) (5, 0, 1) and (4, 1, 1) intersects the x-axis at

Options:

A. $(3, 0, 0)$

B. $(-3, 0, 0)$

C. $(0, 0, 0)$

D. $(1, 0, 0)$

E. $(-1, 0, 0)$

Answer: A

Solution:

Solution:

$\begin{vmatrix} x-2 & y-1 & z-0 \\ 3 & -1 & 1 \\ -1 & 1 & \end{vmatrix} = 0$

$\Rightarrow (x-2)(-1) + (y-1)(1) + (z-0) = 0$
$\Rightarrow -x + 2 - y + 1 + 2z = 0$
$\Rightarrow x + y - 2z - 3 = 0$
When $y = z = 0$, $x = 3$

Question 120

Suppose a line parallel to $ax + by = 0$ (where $b \neq 0$) intersects $5x - y + 4 = 0$ and $3x + 4y - 4 = 0$, respectively, at P and Q. If the midpoint of PQ is (1, 5), then the value of a is $\bar{b}$

Options:

A. $\frac{107}{3}$

B. $-\frac{107}{3}$

C. $\frac{3}{107}$

D. $-\frac{3}{107}$

E. 1

Answer: B

Solution:

Solution:

$(15) = \left[\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)\right] = (2, 5)$

$\frac{x_1+x_2}{2} = 1$

$\Rightarrow x_2 = 2 - x_1$

$\frac{y_1+y_2}{2} = 5$

$\Rightarrow y_2 = 10 - y_1$

Now $Q(x_2, y_2)$ satisfies the equations

$3x + 4y - 4 = 0$

i.e, $3x_2 + 4y_2 - 4 = 0$

i.e, $3[2 - x_1] + 4[10 - y_1] - 4 = 0$

$6 - 3x_1 + 40 - 4y_1 - 4 = 0$

$3x_1 + 4y_1 = 42 ----(1)$

$P(x_1, y_1)$ satisfies the equation

$5x - y + 4 = 0$

i.e, $5x_1 - y_1 = -4$------(2)

solving (1) and (2)

$3x_1 + 4y_1 = 42$

$5x_1 - y_1 = -4$

$20x_1 - 4y_1 = -16$

(1) + (2)$\Rightarrow$

$23x_1 = 26$

$x_1 = \frac{26}{23}$

$\therefore$ from (2)

$y_1 = 5x_1 + 4$

$\equiv \frac{5(26)}{23} + 4$

$\frac{(130 + 92)}{23}$

$\frac{222}{23}$

Now slope of the curve ax + by = 0 is

$y' = \frac{-a}{b}$

Slope of line joining PQ is

$\frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{10 - y_1 - y_1}{2 - x_1 - x_1}$

$= \frac{10 - 2y_1}{2 - 2x_1} = \frac{10 - \frac{222}{23}}{2 - 2\left(\frac{26}{23}\right)}$

$= \frac{230 - 444}{46 - 52} = \frac{-a}{b}$

$$\frac{a}{b} = \frac{444-230}{-6}$$

$$\equiv \frac{214}{-6}$$

$$\frac{-107}{3}$$
