FINAL JEE–MAIN EXAMINATION – APRIL, 2023

(Held On Thursday 06 April, 2023) TIME : 3 : 00 PM to 6 : 00 PM

MATHEMATICS

SECTION–A

১. Three dice are rolled. If the probability of getting different numbers on the three dice is $\frac{p}{q}$ where p and q are co–prime, then q – p is equal to

(১) ৫

(২) ৩

(৩) ১

(৪) ২

Official Ans. by NTA (১)

Sol. Total number of ways = ৬৩ = ২১৬

Favourable outcomes ৬p৩ = ১২০

$\therefore$ Probability = $\frac{১২০}{২১৬} = \frac{৫}{৯}$

$\therefore$ p = ৫, q = ৯

$\therefore$ q – p = ৪

২. Among the statements :

(S১) : ২০২৩২০২২ – ১৯৯৯২০২২ is divisible by ৮.

(S২) : ১৩(১৩)n – ১১n – ১৩ is divisible by ১৪৪ for infinitely many n Î N.

(১) both (S১) and (S২) are incorrect

(২) only (S২) is correct

(৩) both (S১) and (S২) are correct

(৪) only (S১) is correct

Official Ans. by NTA (৩)

Sol. $S_1 = (১৯৯৯ + ২৪)^{২০২২} - (১৯৯৯)^{২০২২}$

$\therefore$ $^{২০২২}C_১(১৯৯৯)^{২০২১}(২৪) + {}^{২০২২}C_২(১৯৯৯)^{২০২০}(২৪)^২ + \ldots$ soon

$S_১$ is divisible by ৮

$S_২$: ১৩(১৩n) – ১১n – ১৩

১৩^n = (১ + ১২)^n = ১ + ১২n + $^nC_২$ ১২^২ + $^nC_৩$ ১২^৩ + ...

১৩(১৩)n – ১১n – ১৩ = ১৪৪n + $^nC_২$ ১২^২ + $^nC_৩$ ১২^৩ + ...

If (n = ১৪৪m, mÎN), then it is divisible by ১৪৪

For infinite value of n.

$\lim_{n\to\infty}\left(\frac{১}{২২} - \frac{১}{২৩} + \frac{১}{২২} - \frac{১}{২৫} + \frac{১}{২৫} - \frac{১}{২৮} + \ldots + \frac{১}{২২n+১} - \frac{১}{২২n+১৪}\right)$

is equal to

(১) $\frac{১}{২}$

(২) ১

(৩) $\frac{২}{৩}$

(৪) ০

Official Ans. by NTA (৪)

Sol. $\left(\frac{১}{২২} - \frac{১}{২৩}\right) + \left(\frac{১}{২২} - \frac{১}{২৫}\right) + \left(\frac{১}{২৫} - \frac{১}{২৮}\right) + \ldots + \left(\frac{১}{২২n+১} - \frac{১}{২২n+১৪}\right)$

$\frac{১}{২}\left(\frac{১}{২২} - \frac{১}{২২n+১৪}\right)$

$\frac{১}{২২} - \frac{১}{২২n+১৪} < ০$

$\lim_{n\to\infty}\left(\frac{১}{২২} - \frac{১}{২২n+১৪}\right) = ০$

$\therefore \lim_{n\to\infty} L = ০$

৪. Let a, b be two non–zero real numbers. Then the number of elements in the set $X = \{z \in C : Re(az^২ + bz) = a \text{ and } Re(az) = b\}$ is equal to

(১) ১

(২) ৩

(৩) ০

(৪) ২

Official Ans. by NTA (৩)

Sol. $Re(az^২ + bz) = a$

$az^2 + bz + az^2 + bz = 2a$

$a(z2 + z^2) + b(z + z) = 2a$ ....(১)

$Re(bz^২ + az) = b$

$bz^2 + az + bz^2 + az = 2b$

$b(z2 + z^2) + a(z + z) = 2b$ ....(২)

(১) × b – (২) × (a)

$\therefore (b^২ - a^২)(z+z) = ০$

Þ (z+z)= ·(a² ¹ b²)

(¹) × a − (²) × (b)

Þ (a² − b²)(z+z)= ²(a² − b²) (a²¹ b²)

z²+z² = ²

Þ (z+z)2 − 2zz = 2

zz= − ¹

Þ ¹ + ¹² = − ¹

Þ No solution

But when a = − b ،

Re(az2-az)=a

Þ Re(a(x2 − y2 + i2xy) - a(x + iy)) = a

Þ a(x2 -y2)-ax = a

Þ x2-y2-x=1

Þ x2-x-1=y 2

For any real values of y there two values of x ،
hence infinite complex numbers are possible .

٥. Let the sets A and B denote the domain and range respectively of the function f(x) = $\frac{¹}{\sqrt{éxù - x}}$ ،
where éêxùú denotes the smallest integer greater than or equal to x . Then among the statements
(S¹) : A Ç B = (¹ ، ¥) − N and
(S²) : A È B = (¹ ، ¥)
(¹) only (S¹) is true
(²) both (S¹) and (S²) are true
(³) neither (S¹) nor (S²) is true
(٤) only (S²) is true

Official Ans . by NTA (¹)

Sol . f(x) = $\frac{¹}{\sqrt{éxù - x}}$

If x Î I éxùú = x (greatest integer function)

If x Ï I êxùú = x + ¹

Þ f(x) = $\begin{cases} \frac{¹}{\sqrt{x - x}}, & x \notin I \\ \frac{¹}{\sqrt{x + ¹ - x}}, & x \in I \\ \frac{¹}{\sqrt{-x}}, & x \in I, \text{(does not exist)} \end{cases}$

Þ domain of f(x) = R − I

Now ، f(x) = $\frac{¹}{\sqrt{¹ - x}}$ ، x Ï I

Þ 0 < x < ¹

Þ 0 < $\sqrt{¹ - x}$ < ¹

Þ $\frac{¹}{\sqrt{¹ - x}}$ > ¹

Þ Range (¹ ، ¥)

Þ A = R − I

B = (¹ ، ¥)

So ، A Ç B = (¹ ، ¥) - N

A È B¹ (¹ ، ¥)

Þ S¹ is only correct

٦. If the solution curve f(x ، y) = · of the differential equation (¹+log ex) $\frac{dx}{dy}$ −xlog e x = y e ، x < · ،
passes through the points (¹ ، ·) and (a ، ²) then aa is equal to

(¹) e²e$\sqrt{²}$

(²) e$\sqrt{²e²}$

(³) e^{e²}

(٤) e^{²e²}

Official Ans . by NTA (٤)

Sol . (¹+ lnx) $\frac{dx}{dy}$ −xlnx=ey

Let x lnx = t

(¹ + ln x) $\frac{dx}{dy}$ = $\frac{dt}{dy}$

−t = ey

$\frac{dt}{dy}$

If = e$^{\int dy}$ = e − y

t.e − y = ∫e − ydy + c

te − y = y c

xlnx e^{y} = y + c

xlnx = ye^{y} + ce^{y}

(¹ ، ·) · = C

Þ xlnx = ye^{y}

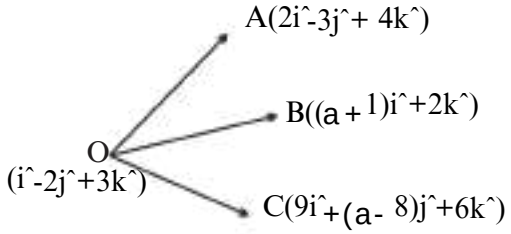
Þ a lna = ²e^{²}

a^{a} = e$^{²e^{²}}$

٧. The sum of all values of α for which the points whose position vectors $\hat{i}+\hat{j}+3\hat{k}$, $2\hat{i}-3\hat{j}+4\hat{k}$, $(\alpha+1)\hat{i}+2\hat{k}$ and $9\hat{i}+(\alpha-8)\hat{j}+6\hat{k}$ are coplanar, is equal to (١) ٦ (٢) ٤ (٣) −٢ (٤) ٢

Official Ans. by NTA (٤)

Sol.



$\vec{OA}\cdot\vec{OB}\times\vec{OC}=0$

$$\begin{vmatrix} 1 & -1 & 1 \\ a & 2 & -1 \\ 8 & a-6 & 3 \end{vmatrix}=0$$

$\Rightarrow a^2-2a-8=0$

$\Rightarrow (a-4)(a+2)=0$

$\therefore a=4,-2$

٨. For the system of equations

$x+y+z=6$

$x+2y+az=10$

$x+3y+5z=b$, which one of the following is NOT true?

(١) System has a unique solution for $a=3$, $b\neq 14$.

(٢) System has no solution for $a=3$, $b=24$.

(٣) System has a unique solution for $a=-3$, $b=14$.

(٤) System has infinitely many solutions for $a=3$, $b=14$.

Official Ans. by NTA (١)

Sol. $x+y+z$

$x+2y+az=10$

$x+3y+5z=b$

$$D=\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & a \\ 1 & 3 & 5 \end{vmatrix}=1(10-3a)-1(5-a)+1(3-z)$$

$=10-3a-5+a+1$

$=6-2a$

For unique solution $6-2a\neq 0 \Rightarrow a\neq 3$

٩. The area bounded by the curves $y=|x-1|+|x-2|$ and $y=3$ is equal to (١) ٣ (٢) ٤ (٣) ٥ (٤) ٦

Official Ans. by NTA (٢)

Sol. $y=|x-1|+|x-2|$ and $y=3$

Required area $=\frac{1}{2}(1+3)\times 2=4$

١٠. Let P be a square matrix such that $P^2=I-P$. For $\alpha, \beta, \gamma, \delta \in N$, if $P^8+P^6=\gamma I-29P$ and $P^8-P^6=\delta I-13P$, then $\alpha+\beta+\gamma-\delta$ is equal to

(١) ١٨

(٢) ٤٠

(٣) ٢٤

(٤) ٢٢

Official Ans. by NTA (٣)

Sol. $P^2=I-P$

$P^8+P^6=\gamma I-29P$, $P^8-P^6=\delta I-13P$

$P^4=(I-P)^2=I-2P+P^2=2I-3P$

$P^6=(2I-3P)(I-P)=5I-8P$

$P^8=(2I-3P)^2=4I-12P+9(I-P)=13I-21P$

$P^8+P^6=18I-29P$

$P^8-P^6=8I-13P$

$\alpha=8;\ \beta=6;\ \gamma=18,\ \delta=8$

$\alpha+\beta+\gamma-\delta=8+6+18-8=24$

١١. All the letters of the word PUBLIC are written in all possible orders and these words are written as in a dictionary with serial numbers. Then the serial number of the word PUBLIC is

(١) ٥٨٠

(٢) ٥٨٢

(٣) ٥٧٨

(٤) ٥٧٦

Official Ans. by NTA (٢)

Sol. B ..... $\rightarrow 5!=120$

C ..... $\rightarrow 5!=120$

I ..... $\rightarrow 5!=120$

L ..... $\rightarrow 5!=120$

PB ..... $\rightarrow 4!=24$

PC ..... $\rightarrow 4!=24$

PI ..... $\rightarrow 4!=24$

PL ..... $\rightarrow 4!=24$

BC ..... $\rightarrow 2!=2$

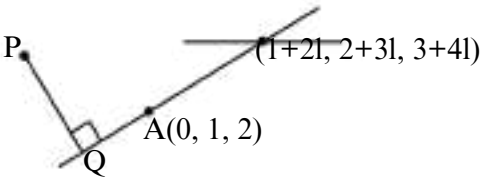
P È BI ... . ® ২! = ২
P È BLC ... . ® ১! = ১
P È BLIC ... . ® = ১
Serial number = ৪(১২০) + ৪(২৪) + ৬ = ৫৮২

১২. Let the line L pass through the point (০, ১, ২), intersect the line $\frac{x-১}{২}=\frac{y-২}{৩}=\frac{z-৩}{৪}$ and be parallel to the plane ২x + y − ৩z = ৪. Then the distance of the point P(l, −৯, ২) from the line L is

(১) ৯ (২) $\sqrt{৫৪}$
(৩) $\sqrt{৬৯}$ (৪) $\sqrt{৭৪}$

Official Ans. by NTA (৪)

Sol.



$\overrightarrow{AB}\cdot\vec{n}=0$

$(2i+3j+4k)\cdot(2i+3k)=0$

২ + ৪l + ১ + ৩l − ৩ − ১২l = ০

৫l = ০ Þ l = ০

Line $\vec{r}=j+2k+m(i+j+k)$

General form : Q(μ, ১ + μ, ২ + μ)

$\overrightarrow{PQ}\cdot\overrightarrow{AB}=0$

(μ − ১) + (১০ + μ) + m = ০

৩m = − ৯ Þ μ = − ৩

distance = $\sqrt{৯+৪৯+১৬}=\sqrt{74}$

১৩. A plane P contains the line of intersection of the planes $\vec{r}\cdot(i+j+k)=৬$ and $\vec{r}\cdot(২i+৩j+৪k)=-৫$. If P passes through the point (০, ২, −২), then the square of distance of the point (১২, ১২, ১৮) from the plane P is

(১) ১২৪০ (২) ৬২০
(৩) ৩১০ (৪) ১৫৫

Official Ans. by NTA (২)

Sol. Equation of plane P is

$(x+y+z-6)+l(2x+3y+4z+5)=0$

Plane passes through the point (০, ২, −২)

$\backslash(2-2-6)+l(6-8+5)=0$

$-6+l(3)=0$

$l=2$

Equation of plane p is

$(x+y+z-6)+2(2x+3y+4z+5)=0$

$5x+7y+9z+4=0$

$d=\left|\frac{5'12+7'12+9'18+4}{\sqrt{52+72+92}}\right|$

$d=\left|\frac{60+84+162+4}{\sqrt{25+49+81}}\right|$

$d=\frac{310}{\sqrt{155}}$

$d2=\frac{310'310}{155}=620$

১৪. Let f(x) be a function satisfying f(x) + f(p − x) = p২, "xÎR. Then $\int_0^p f(x)\sin x\,dx$ is equa

(১) $\frac{p2}{2}$ (২) $\frac{p^২}{২}$
(৩) ২p (৪) $p^২$

Official Ans. by NTA (৪)

Sol. $f(x)+f(p-x)=p2$

$I=\int_0^p f(x)\sin x\,dx$

Applying King's Rule

$I=\int_0^p f(p-x).\sin(p-x)dx$

$2I=\int_0^p[f(x)+f(p-x)]\sin x\,dx$

$2I=\int_0^p p^2\sin x\,dx$

$2I=p2.\int_0^p\sin x\,dx$

$2I=p2'2$

$I=p2$

১৫. If the coefficients of $x^{৭}$ in $\left(ax^২+\frac{১}{২bx}\right)^{১১}$ and $x^{-৭}$ in $\left(ax-\frac{১}{৩bx^২}\right)^{১১}$ are equal, then

(১) ৬৪ab = ২৪৩ (২) ৭২৯ab = ৩২
(৩) ২৪৩ab = ৬৪ (৪) ৩২ab = ৭২৯

Official Ans. by NTA (২)

Sol. $\left(ax^2+\frac{1}{2bx}\right)^{11}$

$T_{r+1}={}^{11}C_r(ax^2)^{11-r}\left(\frac{1}{2bx}\right)^r$

$={}^{11}C_r a^{11-r}\left(\frac{1}{2b}\right)^r x^{22-2r}+{}^{11}C_r a^{11-r}\left(\frac{1}{2b}\right)^r x^{22-3r}$

$22-3r=7$

$3r=15$

$r=5$

Again $\left(ax-\frac{1}{3bx^2}\right)^{11}$

$T_{r+1}={}^{11}C_r(ax)^{11-r}\left(-\frac{1}{3bx^2}\right)^r$

$={}^{11}C_r a^{11-r}\left(\frac{1}{3b}\right)^r x^{11-r-2r}$

$11-3r=7$

$3r=18$

$r=6$

Now $\frac{{}^{11}C_5 a^5}{32b^5}=\frac{{}^{11}C_6 a^6}{36b^6}$

$729ab=32$

١٦. Among the statements

(S١) : $(p\to q)\vee((\sim p)\wedge q)$ is a tautology

(S٢) : $(q\to p)\to((\sim p)\wedge q)$ is a contradiction

(١) neither (S١) and (S٢) is True

(٢) only (S١) is True

(٣) only (S٢) is True

(٤) both (S١) and (S٢) are True

Official Ans. by NTA (١)

Sol. $(p\to q)\vee((\sim p)\wedge q)$

p	q	$p\to q$	$\sim p\wedge q$	$(p\to q)\vee(\sim p)\wedge q)$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	F	T

Not a tautology

p	q	$q\to p$	$(\sim p)\wedge q$	$(q\to p)\vee(\sim p)\wedge q)$
T	T	T	F	F
T	F	T	F	F
F	T	F	T	T
F	F	T	F	F

Not a contradiction

١٧. If the tangents at the points P and Q on the circle $x^2+y^2-2x+y=5$ meet at the point $R\left(\frac{9}{4},2\right)$ then the area of the triangle PQR is

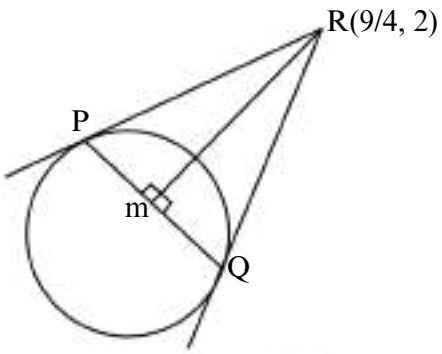
(١) $\frac{13}{4}$ (٢) $\frac{13}{8}$

(٣) $\frac{5}{4}$ (٤) $\frac{5}{8}$

Official Ans. by NTA (٤)

Sol. Equation of circle is $x^2+y^2-2x+y-5=0$

$R=\frac{5}{2}$



Length of $PR=QR=\sqrt{S1}$

$=\sqrt{\frac{81}{16}+4-\frac{2\cdot 9}{4}+2-5}=\frac{5}{4}$

Area of triangle $PQR=\frac{RL^3}{R^2+L^2}=\frac{\frac{5}{2}\cdot\frac{125}{64}}{\frac{25}{4}+\frac{25}{16}}=\frac{5}{8}$

١٨. Let the vectors $\vec{a},\vec{b},\vec{c}$ represent three coterminous edges of a parallelopiped of volume V. Then the volume of the parallelopiped whose coterminous edges are represented by $\vec{a}+\vec{b}+\vec{c}$ and $\vec{a}+2\vec{b}+3\vec{c}$ is equal to

(١) 3V (٢) 6V

(٣) V (٤) 2V

Official Ans. by NTA (٣)

Sol. $V=[\vec{a}\,\vec{b}\,\vec{c}]$

$[\vec{a}+\vec{b}+\vec{c},\vec{b}+\vec{c},\vec{a}+2\vec{b}+3\vec{c}]$

$=\begin{vmatrix}1&0&0\\0&1&1\\1&2&3\end{vmatrix}[\vec{a}\,\vec{b}\,\vec{c}]=1(3-2)V=V.$

١٩. If gcd (m، n) = ١ and ١٢ − ٢٢ + ٣٢ − ٤٢ + ... + (٢٠٢١)٢ − (٢٠٢٢)٢ + (٢٠٢٣)٢ = ١٠١٢ m٢n، then m٢ − n٢ is equal to (١) ٢٠٠ (٣) ٢٢٠

(٢) ٢٤٠

(٤) ١٨٠

Official Ans. by NTA (٢)

Sol. ١٢−٢٢+٣٢−٤٢+... .(٢٠٢١)٢−(٢٠٢٢)٢+(٢٠٢٣)٢=١٠١٢

m٢n

=(١−٢)(١+٢)+(٣−٤)(٣+٤)+... .+(٢٠٢١−٢٠٢٢)(٢٠٢١ +٢٠٢٢) + (٢٠٢٣)٢

=(−١)(١+٢+٣+٤+... . .+٢٠٢٢)+(٢٠٢٣)٢

$$=(-1)\times\frac{(2022)(2023)}{2}+(2023)2$$

=٢٠٢٣(٢٠٢٣−١٠١١) = ٢٠٢٣ × ١٠١٢

m٢n=٢٠٢٣ = ١٧٢ . ٧

m = ١٧، n = ٧

m٢−n٢=١٧٢−٧٢=٢٤٠

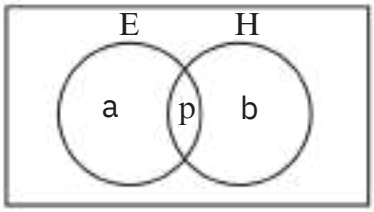
٢٠. In a group of ١٠٠ persons ٧٥ speak English and ٤٠ speak Hindi. Each person speaks at least one of the two languages. If the number of persons، who speak only English is a and the number of persons who speak only Hindi is b، then the eccentricity of the ellipse ٢٥(b٢x٢+a٢y٢)=a٢b٢ is

(١) $\frac{3\sqrt{15}}{12}$ (٢) $\frac{\sqrt{117}}{12}$

(٣) $\frac{\sqrt{119}}{12}$ (٤) $\frac{\sqrt{129}}{12}$

Official Ans. by NTA (٣)

Sol.



a+p=75 ... .(١)

b+p=40 ... .(٢)

a+b+p=100 ... .(٣)

From (١)، (٢) and (٣)

P = ١٥، a=٦٠ and b=٢٥

Now equation of ellipse : ٢٥$\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)=1$

$\frac{x2}{144}+\frac{y2}{25}=1$

Þ e = $\frac{\sqrt{119}}{12}$

SECTION-B

٢١. Let f(x) = $\frac{x}{(1+xn)^{1/n}}$، x Î R − {−١}، n Î N، n < ٢.

If $f^{n}(x)$ = (fofof ... ... upto n times) (x)، then

$\lim_{n\to\infty}\int_0^1 x^{n-2}(f_n(x))dx$ is equal t

Official Ans. by NTA (٠)

Sol. Let $f(x)=\frac{x}{(1+xn)^{1/n}}, x\in R-\{-1\}, n\in N, n>2$

Fn(x) = (fofof ... upto n times)(x)،

then $\lim_{n\to 0}\int x^{n-2}(f_n(x))dx$

$f(f(x))=\frac{x}{(1+2xn)^{1/n}}$

$f(f(f(x)))=\frac{x}{(1+3x^n)^{1/n}}$

Similarly $f_n(x)=\frac{x}{(1+n\times x^n)^{1/n}}$

Now $\lim_{n\to\infty}\int\frac{x^{n-2}}{(1+n\times x^n)^{1/n}}dx=\lim_{n\to\infty}\int\frac{x^{n-1}\times dx}{(1+nx^n)^{1/n}}$

Now ١ + nxn = t

n2×xn-1dx=dt

$x^{n-1}dx=\frac{dt}{n2}$

Þ $\lim_{n\to\infty}\frac{1}{n2}\int_1^{1+n}\frac{dt}{t^{1/n}}$

Þ $\lim_{n\to\infty}\frac{1}{n2}\left[\frac{t^{1-\frac{1}{n}}}{1-\frac{1}{n}}\right]_1^{1+n}$

Þ $\lim_{n\to\infty}\frac{1}{n(n-1)}\left[(1+n)^{\frac{n-1}{n}}-1\right]$ Now let $n=\frac{1}{h}$

Þ $\lim_{h\to 0}\frac{(1+h)^{1-h}-1}{h(1-h)}$

Using series expansion.

Þ 0

২২. The value of $\tan ৯^\circ - \tan ২৭^\circ - \tan ৬৩^\circ + \tan ৮১^\circ$ is _____.

Official Ans. by NTA (৪)

Sol. The value of $\tan ৯^\circ - \tan ২৭^\circ - \tan ৬৩^\circ + \tan ৮১^\circ$

$\Rightarrow \tan ৯^\circ + \cot ৯^\circ - \tan ২৭^\circ - \cot ২৭^\circ$

$\Rightarrow \frac{2}{\sin 18^\circ} - \frac{2}{\sin 54^\circ}$

$\Rightarrow \frac{2 \times 4}{\sqrt{5}-1} - \frac{2 \times 4}{\sqrt{5}+1}$

$\Rightarrow 4$

২৩. If the lines $\frac{x-১}{২} = \frac{২-y}{-৩} = \frac{z-৩}{a}$

and $\frac{x-৪}{৫} = \frac{y-১}{২} = \frac{z}{b}$ intersect,

then the magnitude of the minimum value of ab is _____.

Official Ans. by NTA (১৮)

Sol. If the lines $\frac{x-1}{2} = \frac{2-y}{-3} = \frac{z-3}{a}$

And $\frac{x-4}{5} = \frac{y-}{2} = \frac{z}{b}$ intersect

Point on first line (১, ২, ৩) and point on second line (৪, ১, ০).

Vector joining both points is $-৩\hat{i}+\hat{j}+৩\hat{k}$

Now vector along first line is $২\hat{i}+৩\hat{j}+a\hat{k}$

Also vector along second line is $৫\hat{i}+২\hat{j}+b\hat{k}$

Now these three vectors must be coplanar

$\Rightarrow \begin{vmatrix} 2 & 3 & a \\ 5 & 2 & b \\ -3 & 1 & 3 \end{vmatrix} = 0$

$\Rightarrow 2(6-b) - 3(15+3b) + a(11) = 0$

$\Rightarrow a-b=3$

Now $a=৩+b$

Given expression $8(3+b)\times b = 8\left(b^2+3b\right)$

$= 8\left(b^2+3b+\frac{9}{4}-\frac{9}{4}\right) = 8\left(b+\frac{3}{2}\right)^2 - 18$

So magnitude of minimum value = ১৮

২৪. If $(২০)১৯ + ২(২১)(২০)১৮ + ৩(২১)২(২০)১৭ + \ldots\ldots + ২০(২১)১৯ = k(২০)১৯$, then k is equal to _____

Official Ans. by NTA (৪০০)

Sol. If $(২০)১৯ + ২(২১)(২০)১৮ + ৩(২১)২(২০)১৭ + \ldots + ২০(২১)^{১৯} = k(২০)১৯$ then k is

$20^{19}\left(1+2\times\frac{21}{20}+3\left(\frac{21}{20}\right)^2+\ldots.+20\left(\frac{21}{20}\right)^{19}\right) = k(20)^{19}$

$\Rightarrow k = 1+2\times\frac{21}{20}+3\left(\frac{21}{20}\right)^2+\ldots.+20\left(\frac{21}{20}\right)^{19} \quad \ldots(১)$

$\Rightarrow \frac{k}{20} = \frac{1}{20}+2\left(\frac{21}{20}\right)^2+\ldots. + 19\left(\frac{21}{20}\right)^{19} + 20\left(\frac{21}{20}\right)^{20} \quad \ldots(২)$

Subtracting equation (২) from (১)

$\Rightarrow \left(1-\frac{1}{20}\right)k = 1+\frac{21}{20}+\left(\frac{21}{20}\right)^2+\ldots.+\left(\frac{21}{20}\right)^{19} - 20\left(\frac{21}{20}\right)^{20}$

$\Rightarrow \frac{19}{20}k = \frac{\left(\frac{21}{20}\right)^{20}-1}{\frac{21}{20}-1} - 20\left(\frac{21}{20}\right)^{20}$

$\Rightarrow \frac{k}{20} = \left(\frac{21}{20}\right)^{20} - 1 - 20\left(\frac{21}{20}\right)^{20}$

$\Rightarrow \frac{k}{20} = -20\left(\frac{21}{20}\right)^{20} - 20$

$\Rightarrow k = 400$

২৫. The number of ৪-letter words, with or without meaning, each consisting of ২ vowels and ২ consonants, which can be formed from the letters of the word UNIVERSE without repetition is _____

Official Ans. by NTA (৪৩২)

Sol. UNIVERSE

Vowels: E, I, U

Consonants: N, V, R, S

$\Rightarrow {}^3C_2 \times {}^4C_2 \times ৪! = ৩ \times ৬ \times ২৪ = ৪৩২$

২৬. The number of points, where the curve $y = x^৫ - ২০x^৩ + ৫০x + ২$ crosses the x-axis, is _____.

Official Ans. by NTA (৫)

Sol. $y = x^5 - 20x^3 + 50x + 2$

$\frac{dy}{dx} = 5x^4 - 60x^2 + 50 = 5(x^4 - 12x^2 + 10)$

$\frac{dy}{dx} = 0 \Rightarrow x^4 - 12x^2 + 10 = 0$

$\Rightarrow x^2 = \frac{12 \pm \sqrt{144 - 40}}{2}$

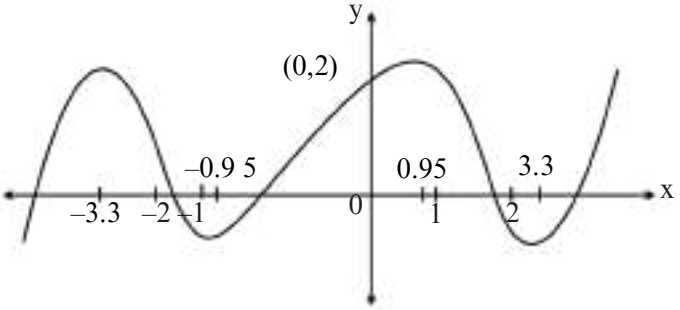
$\Rightarrow x = \frac{6 \pm \sqrt{26}}{2} \Rightarrow x^2 \approx 6 \pm 5.1$

$\Rightarrow x \approx 11.1, 0.9$

$\Rightarrow x \approx \pm 3.3, \pm 0.95$

$f(0) = 2,\ f(1) = +ve,\ f(2) = -ve$

$f(-1) = -ve,\ f(-2) = +ve$



২৭. For $a, b, z \in C$ and $\lambda < 1$, if $\sqrt{\lambda - 1}$ is the radius of the circle $|z-a|^2 + |z-b|^2 = 2\lambda$, then $|a-b|$ is equal to _____.

Official Ans. by NTA (২)

Sol. For circle :

$|z - z_1|^2 + |z - z_2|^2 = |z_1 - z_2|^2$

$r = \frac{|z_1 - z_2|}{2} = \frac{|\alpha - \beta|}{2} = \sqrt{\lambda - 1}$

$2\lambda = |\alpha - \beta|^2$

$|\alpha - \beta| = 2\sqrt{\lambda - 1}$

$|\alpha - \beta|^2 = 4\lambda - 4 = 2\lambda$

$\lambda = 2$

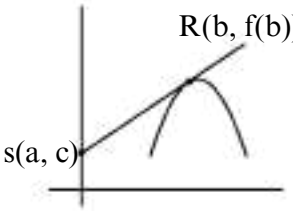
$\Rightarrow |\alpha - \beta|^2 = 4$

$|\alpha - \beta| = 2$

২৮. Let a curve $y = f(x), x \in (0, \infty)$ pass through the points $P\left(1, \frac{3}{2}\right)$ and $Q\left(a, \frac{1}{2}\right)$. If the tangent at any point $R(b, f(b))$ to the given curve cuts the y-axis at the point $S(0, c)$ such that $bc = 3$, then $(PQ)^2$ is equal to _____.

Official Ans. by NTA (৫)

Sol.



Equation of tangent at $R(b, f(2))$ is

$y - f(b) = f'(b).(x - b)$

which passes through $(0, c)$

$\Rightarrow c - f(b) = f'(b).(-b)$

$\Rightarrow \frac{3}{b} - f(b) = f'(b).(-b)$

$\Rightarrow bf'(b) - f(b) = -\frac{3}{b}$

$\Rightarrow \frac{bf'(b) - f(b)}{b^2} = -\frac{3}{b^3}$

$\Rightarrow d\left(\frac{f(b)}{b}\right) = -\frac{3}{b^3} \Rightarrow \frac{f(b)}{b} = \frac{3}{2b^2} + \lambda$

Which passes through $(1, 3/2)$

$\Rightarrow \frac{3}{2} = \frac{3}{2} + \lambda \Rightarrow \lambda = 0$

$\Rightarrow f(b) = \frac{3}{2b}$

$f(a) = \frac{1}{2} \Rightarrow \frac{1}{2} = \frac{3}{2b} \Rightarrow b = 3$

$\Rightarrow c = 1 \Rightarrow Q(3, 1/2)$

$\Rightarrow PQ^2 = 2^2 + (1)^2 = 5$

২৯. Let the eccentricity of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is reciprocal to that of the hyperbola $2x^2 - 2y^2 = 1$. If the ellipse intersects the hyperbola at right angles, then square of length of the latus-rectum of the ellipse is _____.

Official Ans. by NTA (২)

Sol. $eH=\sqrt{2}$

$eE=\frac{1}{\sqrt{2}}$

Since the curves intersect each other orthogonally

The ellipse and the hyperbola are confocal

$H: \frac{x^2}{1/2}-\frac{y^2}{1/2}=1$

$\therefore$ foci $=(1,0)$

For ellipse $a.eE=1$

$\therefore a=\sqrt{2}$

$(e_E)^2=\frac{1}{2} \therefore 1-\frac{b^2}{a^2}=\frac{1}{2} \therefore \frac{b^2}{a^2}=\frac{1}{2}$

$\therefore b^2=1$

Length of $L.R.=\frac{2b^2}{a}=\frac{2}{\sqrt{2}}=\sqrt{2}$

३०. If the mean and variance of the frequency distribution

xi	2	4	6	8	10	12	14	16
fi	4	4	a	15	8	b	4	5

are ९ and १५.०८ respectively, then the value of a^2+b^2-ab is _____.

Official Ans. by NTA (२५)

Sol.

xi	fi	fixi	fx^2ii
२	४	८	१६
४	४	१६	६४
६		६	३६a
८	१५	१२०	९६०
१०	८	८०	८००
१२	b	१२b	१४४b
१४	४	५६	७८४
१६	५	८०	१२८०

$N=\sum f_i=40+a+b$

$\sum fixi=360+6a+12b$

$\sum fix_i^2=3904+36a+144b$

$\text{Mean}(\bar{x})=\frac{\sum fixi}{\sum fi}=9$

$\therefore 360+6a+12b=9(40+a+b)$

$3a=3b \therefore a=b$

$s^2=\frac{\sum fixi^2}{\sum fi}-\left(\frac{\sum fixi}{\sum fi}\right)^2$

$\therefore \frac{3904+36a+144b}{40+a+b}-(\bar{x})^2=15.08$

$\therefore \frac{3904+180a}{40+2a}-(9)^2=15.08$

$\therefore a=5$

Now, $a^2+b^2-ab=a^2=25$

PHYS ICS

SECTION-A

٣١. A ٢ meter long scale with least count of ٠.٢ cm is
used to measure the locations of objects on an
optical bench. While measuring the focal length of
a convex lens, the object pin and the convex lens
are placed at ٨٠ cm mark and lm mark,
respectively. The image of the object pin on the
other side of lens coincides with image pin that is
kept at ٧٠ cm mark. The % error in the estimation
of focal length is: (١) ١.٠٢ (٢) ٠.٨٥ (٣) ١.٧٠ (٤) ٠.٥١

Official Ans. by NTA (٣)

Sol. Least count = ٠.٢ cm

$u = -100 \pm 0.2 \pm 80 \pm 0.2 \pm 20 \pm 0.4$ cm

$v = 180 \pm 0.2 \pm 100 \pm 0.2 \pm 80 \pm 0.4$ cm

From lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \frac{1}{f} = \frac{1}{80} - \frac{1}{-20}$$

f = ١٦ cm

Also $\frac{\Delta v}{v^2} = \frac{\Delta u}{u^2} + \frac{\Delta f}{f^2}$

$$\frac{\Delta f}{f} \times 100 = \left(\frac{\Delta v}{v^2} + \frac{\Delta u}{u^2}\right) \times f \times 100$$

$$\% f = \left(\frac{0.4}{400} + \frac{0.4}{6400}\right) 16 \times 100$$

= ١.٧٠ A capacitor of capacitance ١٥٠.٠ μF is

٣٢. connected
to an alternating source of emf given by E = ٣٦ sin(١٢٠πt) V. The maximum value of current in the
circuit is approximately equal to:
(١) ٢A (٢) $\frac{1}{\sqrt{2}}$ A

(٣) $\sqrt{2}$A (4) 22A

Official Ans. by NTA (١)

TEST PAPER WITH SOLUTION

Sol. $I_0 = \frac{E_0}{x_c} = \frac{E_0}{\frac{1}{\omega C}} = E_0 \omega C$

$$I_0 = 36 \times 120 \times 150 \times 10 \times 6$$

$$I_0 = 2.03$$

$$\simeq 2\,A$$

٣٣. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A: When you squeeze one end of a tube to get toothpaste out from the other end, Pascal's principle is observed.

Reason R: A change in the pressure applied to an enclosed incompressible fluid is transmitted undiminished to every portion of the fluid and to the walls of its container.

In the light of the above statements, choose the most appropriate answer from the options given below

(١) A is not correct but R is correct

(٢) A is correct but R is not correct

(٣) Both A and R are correct and R is the correct explanation of A

(٤) Both A and R are correct but R is NOT the correct explanation of A

Official Ans. by NTA (٣)

Sol. (R) is the statement of Pascal's principle & which explains the assertion (S)

٣٤. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A: The phase difference of two light waves change if they travel through different media having same thickness, but different indices of refraction.

Reason R: The wavelengths of waves are different in different media.

In the light of the above statements, choose the most appropriate answer from the options given below

(١) Both A and R are correct but R is NOT the correct explanation of A

(٢) A is correct but R is not correct

(٣) Both A and R are correct and R is the correct explanation of A

(٤) A is not correct but R is correct

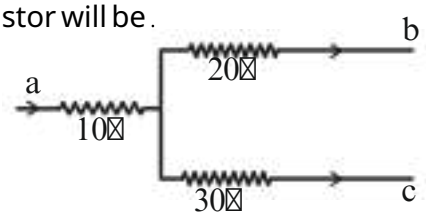
Official Ans. by NTA (٣)

Sol. As medium changes, optical path changes.

Also, $\frac{\Delta x}{\lambda} = \frac{\Delta\phi}{2\pi}$

Hence phase difference changes.

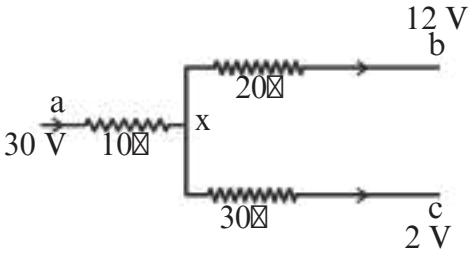
٣٥. Figure shows a part of an electric circuit. The potentials at points a, b and c are ٣٠ V, ١٢ V and ٢V respectively. The current through the ٢٠Ω resistor will be.



(١) ٠.٤ A (٢) ٠.٢ A (٣) ٠.٦ A (٤) ١.٠ A

Official Ans. by NTA (١)

Sol. Sum of current at junction point will be zero :



$$\frac{x-30}{10}+\frac{x-12}{20}+\frac{x-2}{30}=0$$

$$x\left(\frac{1}{10}+\frac{1}{20}+\frac{1}{30}\right)=\frac{30}{10}+\frac{12}{20}+\frac{2}{30}$$

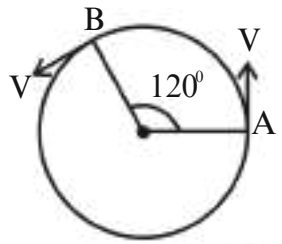
$$x=\frac{6+3+2}{60}=\frac{180+36+4}{60}$$

$$x=\frac{220}{11}=20V$$

Current through $20\Omega=\frac{x-12}{20}$

$$=\frac{20-12}{20}=\frac{2}{5}=0.4A$$

٣٦. As shown in the figure, a particle is moving with constant speed ٦ m/s. Considering its motion from A to B, the magnitude of the average velocity is:



(١) ٦ m/s (٢) $\sqrt{3}$ m/s

(3) $2\sqrt{3}$m/s (٤) $1.5\sqrt{3}$m/s

Official Ans. by NTA (٤)

Sol. $|\langle v\rangle|=\frac{|r_f-r_i|}{\Delta t}$

$$=\frac{2R\cos\left(\frac{180-120}{2}\right)}{\frac{2\pi R}{3v}}=3\cos 30^0$$

$1.5\sqrt{3}$m/s

Correct option is (٤)

٣٧. The work functions of Aluminium and Gold are ٤.١ eV and ٥.١ eV respectively. The ratio of the slope of the stopping potential versus frequency plot for Gold to that of Aluminium is

(١) ١.٢٤ (٢) ٢

(٣) ١ (٤) ١.٥

Official Ans. by NTA (٣)

Sol. $eV_s = k_{max}$

$$V_s = \frac{h}{e}f - \frac{\phi}{e}$$

Slope is independent of nature of metal

$$\text{slope}_{Gold} = \text{slope}_{Aluminium}$$

٣٨. The ratio of speed of sound in hydrogen gas to the speed of sound in oxygen gas at the same temperature is:

(١) ٤ : ١ (٢) ١ : ٢

(٣) ١ : ٤ (٤) ١ : ١

Official Ans. by NTA (١)

Sol. $C = \sqrt{\frac{\gamma RT}{M}}$

$$C = \frac{1}{\sqrt{M}}$$

$$\frac{C_{H_2}}{C_{O_2}} = \sqrt{\frac{32}{2}} = 4:1$$

Correct option (١) A child of mass ٥ kg is going

٣٩. round a merry-go-round that makes ١ rotation in ٣.١٤ s. The radius of the merry-go-round is ٢ m. The centrifugal force on the child will be

(١) ٨٠ N (٢) ٥٠ N

(٣) ١٠٠ N (٤) ٤٠ N

Official Ans. by NTA (٤)

Sol. $\omega = \frac{2\pi}{3.14} = 2 \text{ rad/s}$

$|f_{centrifugal}| = m\omega^2 R$

$= M\omega^2 R$

$= ٤٠ \text{ N}$

Correct option (٤)

٤٠. A particle starts with an initial velocity of ١٠.٠ ms along x-direction and accelerates uniformly at the rate of ٢.٠ ms^{-2}. The time taken by the particle to reach the velocity of ٦٠.٠ ms is_____.

(١) ٦s (٢) ٣s

(٣) ٣٠s (٤) ٢٥s

Official Ans. by NTA (٤)

Sol. $v = u + at$

$٦٠ = ١٠ + ٢t$

$t = ٢٥ \text{ sec}.$

Correct option (٤)

٤١. Choose the incorrect statement from the following:

(١) The speed of satellite in a given circular orbit remains constant.

(٢) For a planet revolving around the sun in an elliptical orbit, the total energy of the planet remains constant.

(٣) When a body fall towards earth, the displacement of earth towards the body is negligible.

(٤) The linear speed of a planet revolving around the sun remains constant.

Official Ans. by NTA (٤)

Sol. Planets revolve in elliptical paths around sun. Thus their linear speed is not constant

٤٢. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R Assertion A: Diffusion current in a p-n junction is greater than the drift current in magnitude if the junction is forward biased. Reason R: Diffusion current in a p-n junction is from the n-side to the p-side if the junction is forward biased. In the light of the above statements, choose the most appropriate answer from the options given below (١) Both A and R are correct and R is the correct explanation of A (٢) Both A and R are correct but R is NOT the correct explanation of A (٣) A is correct but R is not correct (٤) A is not correct but R is correct
Official Ans. by NTA (٣)

Sol. In forward biased condition, diffusion of majority charge carriers takes place from p-side to n-side which constitute the diffusion current.

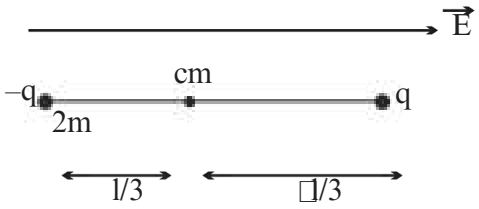
٤٣. A dipole comprises of two charged particles of identical magnitude q and opposite in nature. The mass 'm' of the positive charged particle is half of the mass of the negative charged particle. The two charges are separated by a distance 'l'. If the dipole is placed in a uniform electric field 'E', such a way that dipole axis makes a very small angle with the electric field, 'E'. The angular frequency of the oscillations of the dipole when released is given by:

(١) $\sqrt{\frac{8qE}{3ml}}$ (٢) $\sqrt{\frac{4qE}{ml}}$

(٣) $\sqrt{\frac{4qE}{3ml}}$ (٤) $\sqrt{\frac{8qE}{ml}}$

Official Ans. by NTA (٣)

Sol.



If released, it will oscillate about centre of mass.
For small 'θ'

$\tau = -PE.\theta$

$\Rightarrow \left[2m\frac{l^2}{9} + m\frac{4l^2}{9}\right]\alpha = -qlE\theta$

$\Rightarrow \frac{2ml^2}{3}\alpha = -qlE\theta \quad \Rightarrow \alpha = -\frac{3qE}{2ml}\theta$

$\omega = \sqrt{\frac{3qE}{2ml}}$

٤٤. The energy density associated with electric field E and magnetic field B of an electromagnetic wave in free space is given by (ε_0 – permittivity of free space, μ_0 – permeability of free space)

(١) $U_E = \frac{E^2}{2\varepsilon_0}, U_B = \frac{B^2}{2\varepsilon_0}$

(٢) $U_E = \frac{E^2}{2\varepsilon_0}, U_B = \frac{\varepsilon_0 B^2}{2}$

(٣) $U_E = \frac{\varepsilon_0 E^2}{2}, U_B = \frac{\varepsilon_0 B^2}{2}$

(٤) $U_E = \frac{\varepsilon_0 E^2}{2}, U_B = \frac{B^2}{2\mu_0}$

Official Ans. by NTA (٤)

Sol. $U_E = \frac{1}{2}\varepsilon_0 E^2, U_B = \frac{B^2}{2\mu_0}$

٤٥. The temperature of an ideal gas is increased from ٢٠٠ K to ٨٠٠ K. If r.m.s. speed of gas at ٢٠٠ K is v_0. Then, r.m.s. speed of the gas at ٨٠٠ K will be:

(١) v_0 (٢) $2v_0$

(٣) $\frac{v_0}{4}$ (٤) $4v_0$

Official Ans. by NTA (٤)

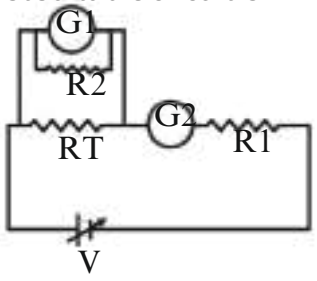
Sol. $V_{rms} = \sqrt{\frac{3RT}{M}}$

$\Rightarrow V_{rms} \propto \sqrt{T}$

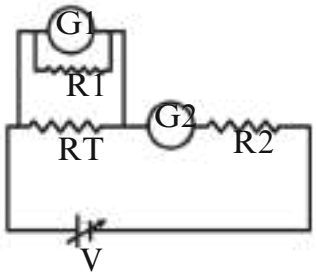
Increasing temperature 4 times, rms speed gets doubled.

٤٦. A student is provided with a variable voltage source V, a test resistor Rr = ١٠Ω, two identical galvanometers Gl and G٢ and two additional resistors, R١ = ١٠MΩ and R٢ = ٠.٠٠١Ω. For conducting an experiment to verify ohms law, the most suitable circuit is:

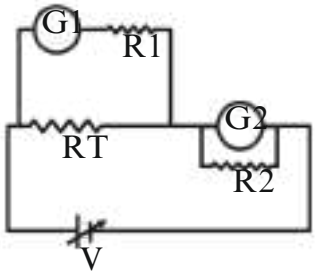
(١)



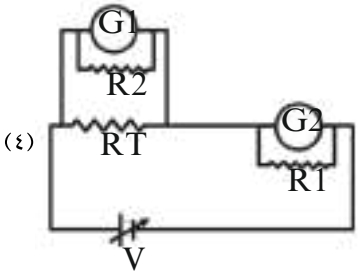
(٢)



(٣)



(٤)



Official Ans. by NTA (٣)

Sol. To convert galvanometer into ammeter low resistances should be added into parallel & for voltmeter conversion, a very high resistance should be added in series.

٤٧. A body cools in ٧ minutes from ٦٠°C to ٤٠°C. The temperature of the surrounding is ١٠°C. The temperature of the body after the next ٧ minutes will be:

(١) ٣٢°C (٢) ٣٠°C

(٣) ٢٨°C (٤) ٣٤°C

Official Ans. by NTA (٣)

Sol. using average rate of Newton's law of cooling

$$\frac{T_1 - T_2}{t} = K\left(\frac{T_1 + T_2}{2} - T_s\right)$$

Given $\frac{60-40}{7} = K\left(50-\frac{40+T}{2}\right)$ ... (i)

& $\frac{40-T}{7} = K\left(\frac{40+T}{2}-10\right)$ ... (ii)

From (i) & (ii)

T = ٢٨°C

٤٨. A small particle of mass m moves in such a way that its potential energy $U = \frac{1}{2}m\omega^2 r^2$ where ω is constant and r is the distance of the particle from origin. Assuming Bohr's quantization of momentum and circular orbit, the radius of n^{th} orbit will be proportional to.

(١) $\sqrt{n}$ (٢) n

(٣) n^2 (٤) $\frac{1}{n}$

Official Ans. by NTA (١)

Sol. $U = \frac{1}{2}m\omega^2 r^2$

$F = -\frac{dv}{dr} = m\omega^2 r$

Now $r = \frac{mv^2}{r} = v = \omega r$ ... (i)

& $mvr = \frac{nh}{2\pi}$ ... (ii)

From (i) & (ii)

$m\omega r^2 = \frac{nh}{2\pi}$

$r \propto \sqrt{n}$

٤٩. For an amplitude modulated wave the minimum amplitude is ٣V، while the modulation index is ٦٠٪. The maximum amplitude of the modulated wave is: (١) ١٥V (٢) ١٢V (٣) ١٠V (٤) ٥V Official Ans. by NTA (٢)

Sol. Modulation index $\frac{A_m}{A_c} = 0.6$

Minimum amplitude of modulated wav

$= A_c - A_m = 3$

$C = 0.6A_c - 3 = 0.4A_c - 3$

$= A$

$A_c = \frac{3}{0.4} = \frac{15}{2} = 7.5V$

$A_m = 0.6A_c = 4.5V$

∴ Maximum amplitude = Ac + Am

= ٧.٥ + ٤.٥ = ١٢V

∴ Correct option is (٢)

٥٠. The weight of a body on the surface of the earth is ١٠٠ N. The gravitational force on it when taken at a height، from the surface of earth، equal to one-fourth the radius of the earth is:

(١) ١٠٠ N

(٢) ٦٤ N

(٣) ٥٠ N

(٤) ٢٥ N

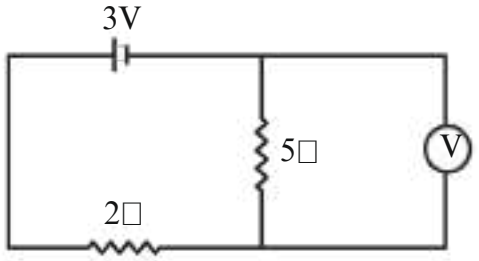
Official Ans. by NTA (٢)

Sol. $g' = \frac{gR^2}{r^2} = \frac{gR^2}{\left(R + \frac{R}{4}\right)^2} = \frac{16g}{25}$

$= \text{Weight} = \frac{16}{25} \times 100 = 64N$

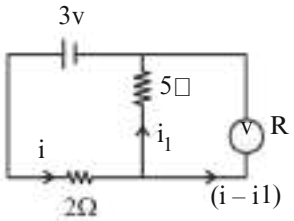
SECTION-B

٥١. As shown in the figure، the voltmeter reads ٢V across ٥ Ω resistor. The resistance of the voltmeter is_____Ω.



Official Ans. by NTA (٢٠)

Sol. $i_1 = \frac{2V}{5\Omega} = \frac{2}{5} A$



$i = \frac{1V}{2\Omega} = \frac{1}{2} A$

∴ Current through voltmeter $I_1 = i - i_1$

$= \frac{1}{2} - \frac{2}{5} = \frac{5-4}{10} = \frac{1}{10} A$

∴ For voltmeter

$2 = \left(\frac{1}{10}\right) R \quad R = 20\Omega$

٥٢. A metal block of mass m is suspended from a rigid support through a metal wire of diameter ١٤ mm. The tensile stress developed in the wire under equilibrium state is ٧×١٠^{٧} Nm$^{-٢}$. The value of mass m is _____ kg.

(Take، g = ٩.٨ ms$^{-٢}$ and $\pi = \frac{22}{7}$)

Official Ans. by NTA (١١)

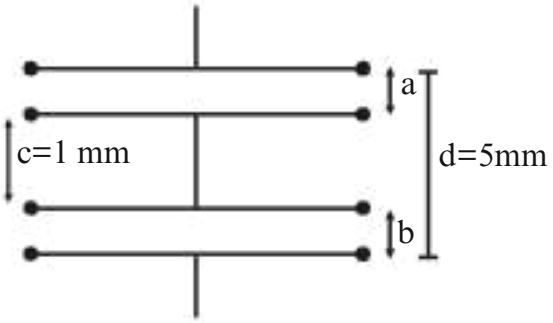
Sol. Tensile stress $= \frac{F}{A} = \frac{4mg}{\pi D^2}$

$m = \frac{\pi D^2}{4g}$

$= \frac{22}{7} \times \frac{14 \times 10 \times 3 \times 7 \times 105}{4 \times 9.8}$

= ١١ kg

٥٣. As shown in the figure, two parallel plate capacitors having equal plate area of ٢٠٠ cm^2 are joined in such a way that a || b. The equivalent capacitance of the combination is x ε$_0$ F. The value of x is_____.



Official Ans. by NTA (٥)

Sol. $C = \frac{\varepsilon_0 A}{d - c}$

$= \frac{\varepsilon_0 \times 200 \times 10^{-4}}{4 \times 10^{-3}}$

$= x \times 5$

٥٤. The situation is equivalent to a conducting slab placed between the plates A ring and a solid sphere rotating about an axis passing through their centers have same radii of gyration. The axis of rotation is perpendicular to the plane of ring. The ratio of radius of ring to that of sphere is $\sqrt{\frac{2}{x}}$. The value of x is _____

Official Ans. by NTA (٥)

Sol. For ring $I = mR^2 = mK_1^2$

$\Rightarrow$ Radius of gyration $K_1 = R_1$

For solid sphere

$I = \frac{2}{5}mR^2 = mK_2^2$

$\Rightarrow$ Its radius of gyration $K_2 = \sqrt{\frac{2}{5}}R$

$\because K_1 = K_2$

$\Rightarrow R_1 = \sqrt{\frac{2}{5}}R$

$\frac{R_1}{R} = \sqrt{\frac{2}{5}}$

$x = 5$

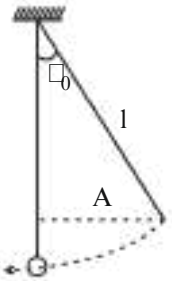
٥٥. A simple pendulum with length ١٠٠ cm and bob of mass ٢٥٠ g is executing S.H.M. of amplitude ١٠cm. The maximum tension in the string is found to be $\frac{x}{40}$ N. The value of x is_____.

Official Ans. by NTA (٩٩)

Sol. $\sin\theta_0 = \frac{A}{l} = \frac{10}{100} = \frac{1}{10}$

From conservation of energy

$\frac{1}{2}mv^2 = mgl(1 - \cos\theta_0)$



Maximum tension occurs at mean position.

$T - mg = \frac{mv^2}{l}$

$T = mg + \frac{mv^2}{l}$

$\Delta T = mg + 2mg(1 - \cos\theta)$

$= mg\left(1 + 2\left(1 - \sqrt{1 - \frac{1}{100}}\right)\right)$

$= mg\left(3 - 2\sqrt{1 - \frac{1}{100}}\right)$

$= \frac{250}{1000} \times 9.8\left(3 - 2\left(1 - \frac{1}{200}\right)\right) = \frac{94}{40}$

$= x = 99$

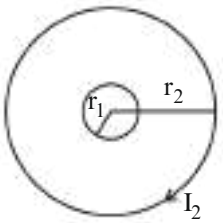
٥٦. Two concentric circular coils with radii ١ cm and ١٠٠٠ cm, and number of turns ١٠ and ٢٠٠ respectively are placed coaxially with centers this coinciding. The mutual inductance of arrangement will be _____ $\times 10^{-8}$ H.

(Take, $\pi^2 = 10$)

Official Ans. by NTA (٤)

Sol. $r_1 = 1\text{cm}, N_1 = 10$

$r_2 = 1000\text{cm}, N_2 = 200$



$\phi_{1,2} = MI_2$

$N_2B_2 = N_1A_1 = MI_2$

$= N_1N_2\frac{\mu_0}{2r_2}\pi r_1^2 = MI_2$

$= M = \frac{10 \times 200 \times 4\pi \times 10^{-7} \times \pi \times (0.01)^2}{2 \times 10}$

$= M = 4 \times 10^{-8}$

٥٧. A beam of light consisting of two wavelengths ٧٠٠٠ Å and ٥٥٠٠ Å is used to obtain interference pattern in Young's double slit experiment. The distance between the slits is ٢.٥ mm and the distance between the plane of slits and the screen is ١٥٠ cm. The least distance from the central fringe, where the bright fringes due to both the wavelengths coincide, is $n \times 10^{-2}$ m. The value of n is_____.

Official Ans. by NTA (٤٦٢)

Sol. $d = ٢.٥$ mm, $D = ١٥٠$ cm

Fringe width $= \frac{\lambda D}{d}$

Let n^{th} bright tringle of λ_1 match with m^{th} bright triangle of λ_2

$n\lambda_1 = m\lambda_2$

$\frac{n}{m} = \frac{\lambda_2}{\lambda_1} = \frac{5500}{7000}$

$\frac{n}{m} = \frac{11}{14}$

Distance where bright fringe will match

$n\lambda_1 = \frac{11 \times 7000\ \text{A}^0 \times 150\ \text{cm}}{0.25\ \text{cm}}$

$= 462 \times 10^{-5}$

٥٨. A body is dropped on ground from a height 'h١' and after hitting the ground, it rebounds to a height 'h٢' If the ratio of velocities of the body just before and after hitting ground is ٤, then percentage loss in kinetic energy of the body is $\frac{x}{4}$. The value of x is_____.

Official Ans. by NTA (٣٧٥)

Sol. Let V१ and V२ are velocity just before and just after hitting the floor.

$$\frac{V_1}{V_2} = \frac{\sqrt{4}}{1} = \frac{4}{V2}$$

$$KE_{before} = \frac{1}{2}mV_1^2$$

$$KE_{after} = \frac{1}{2}mV_2^2 = \frac{1}{2}\frac{m.V_1^2}{16}$$

$$\Delta KE = \frac{1}{2}mV_1^2\left(\frac{1}{16} - 1\right) = -\frac{15}{32}mV_1^2$$

$$\% \text{ change} = \frac{\Delta KE}{KE_{before}} \times 100\%$$

$$= \frac{-15}{16} \times 100 = \frac{-375}{4}\%$$

५९. Experimentally it is found that १२.८ eV energy is required to separate a hydrogen atom into a proton and an electron. So the orbital radius of the electron in a hydrogen atom is $\frac{9}{x} \times ١٠^{-١٠}$ m. The value of the x is _____.

(१eV = १.६ × १०$^{-१९}$ J, $\frac{1}{4\pi\varepsilon_0} = 9 \times 109 Nm^2/C^2$ and electronic charge = १.६ × १०$^{-१९}$ C)

Official Ans. by NTA (१६)

Sol. Binding energy of system $= \frac{ke^2}{2r}$ joule and

$$\frac{ke^2}{2r} = 12.8ev$$

$$\frac{9 \times 109 \times (1.6 \times 10^{-19})^2}{2r} = 12.8 \times 1.6 \times 10^{-19}$$

$$r = \frac{9 \times 109 \times 1.6 \times 10^{-19}}{12.8 \times 2}$$

$$r = \frac{9 \times 10 \times 10}{16}$$

६०. A proton with a kinetic energy of २.० eV moves into a region of uniform magnetic field of magnitude $\frac{\sqrt{2}}{2} \times 10^{-3}$ T. The angle between the direction of magnetic field and velocity of proton is ६०°. The pitch of the helical path taken by the proton is _____ cm.

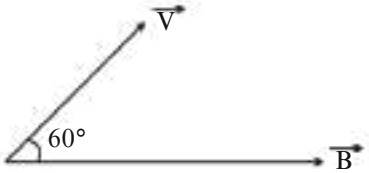
(Take, mass of proton = १.६ × १०$^{-२७}$ kg and Charge on proton = १.६ × १०$^{-१९}$ C).

Official Ans. by NTA (४०)

Sol. $B = \frac{\sqrt{2}}{2} \times 10^{-3}$

$$K.E. = \frac{1}{2}mV^2$$

$$V = \sqrt{\frac{2KE}{m}}$$



Pitch = v cos ६०° × time period of one rotation

$$= v\cos 60^\circ \times \frac{2\pi m}{eB}$$

$$= \sqrt{\frac{2 \times 2 \times 1.6 \times 10^{-19}}{1.6 \times 10 \times 27}} \cos 60^\circ \times \frac{2 \times 1.6 \times 10 \times 27}{1.6 \times 10 \times 9 \times \frac{\sqrt{2}}{2} \times 10^{-3}}$$

$$= 2 \times 104 \times \frac{1}{2} \times 4 \times 10^{-5}$$

$$= 4 \times 10^{-1} m = 40 cm$$

SECTION-A

61. Ion having highest hydration enthalpy among the given alkaline earth metal ions is:-

(1) Be^{2+}

(2) Ba^{2+}

(3) Sr^{2+}

(4) Ca^{2+}

Official Ans. by NTA (1)

Sol. Hydration enthalpy $\propto \frac{1}{\text{size}}$

Down the group as size increases hydration enthalpy decreases

Order : $Be^{2+} > Mg^{+2} > Ca^{+2} > Sr^{+2} > Ba^{+2}$

62. The IUPAC name of $K_3[Co(C_2O_4)_3]$ is :-

(1) Potassium trioxalatocobaltate(III)

(2) Potassium tris(oxalato)cobalt(III)

(3) Potassium tris(oxalato)cobaltate(III)

(4) Potassium trioxalatocobalt(III)

Official Ans. by NTA (1)

Sol. IUPAC name of $K_3[Co(C_2O_4)_3]$ is

Potassium trioxalatocobaltate(III)

63. Match **List I** with **List II**

List I	**List II**
Natural Amino acid	**One Letter Code**
(A) Arginine	(I) D
(B) Aspartic acid	(II) N
(C) Asparagine	(III) A
(D) Alanine	(IV) R

Choose the correct answer from the options given below :-

(1) (A)–IV, (B)–I, (C)–III, (D)–II

(2) (A)–I, (B)–III, (C)–IV, (D)–II

(3) (A)–III, (B)–I, (C)–II, (D)–IV

(4) (A)–IV, (B)–I, (C)–II, (D)–III

Official Ans. by NTA (4)

Sol. Factual.

64. Element not present in Nessler's reagent is:-

(1) Hg

(2) I

(3) K

(4) N

Official Ans. by NTA (4)

Sol. Nessler reagent is – $K_2[HgI_4]$

65. Structure of $BeCl_2$ in solid state, vapour phase and at very high temperature respectively are :-

(1) Dimeric, Polymeric, Monomeric

(2) Polymeric, Dimeric, Monomeric

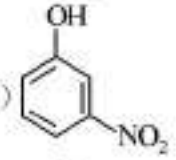
(3) Monomeric, Dimeric, Polymeric

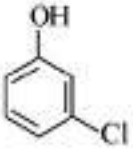
(4) Polymeric, Monomeric, Dimeric

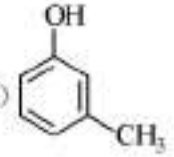
Official Ans. by NTA (2)

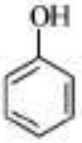
Sol. In solid state $BeCl_2$ as polymer, in vapour state it form chloro-bridged dimer while above 1200K it is monomer.

66. The strongest acid from the following is

(1) 

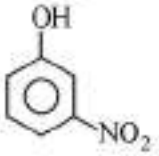
(2) 

(3) 

(4) 

Official Ans. by NTA (1)

Sol. Strongest acid from the following is



$-NO_2$ group has more EWG nature so more acidic.

67. Group-13 elements react with O_2 in amorphous form to form oxides of type M_2O_3 (M = element). Which among the following is the most basic oxide?

(1) Al_2O_3

(2) Ga_2O_3

(3) Tl_2O_3

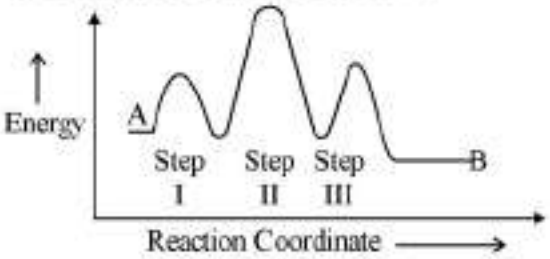
(4) B_2O_3

Official Ans. by NTA (3)

Sol. As electropositive character increases basic character of oxide increases.

$$\underbrace{B_2O_3}_{\text{acidic}} < \underbrace{Al_2O_3 < Ga_2O_3}_{\text{amphoteric}} < \underbrace{In_2O_3 < Tl_2O_3}_{\text{basic}}$$

68. Consider the following reaction that goes from A to B in three steps as shown below:-



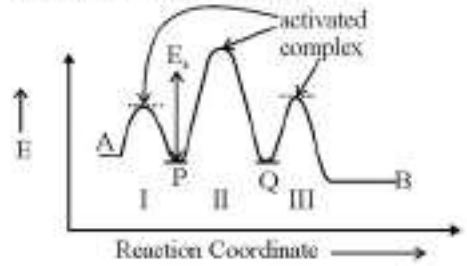
Choose the correct

Number of intermediates	Number of Activated complexes	Rate determining step
(1) 3	2	II
(2) 2	3	II
(3) 2	3	I
(4) 2	3	III

Official Ans. by NTA (2)

Sol. Step with highest activation energy is RDS, so step II is RDS

No. of activated complex = 3



P and Q are intermediates

(Number of intermediates = 2)

69. Given below are two statements : one is labelled as **"Assertion A"** and the other is labelled as **"Reason R".**

Assertion A : In the complex $Ni(CO)_4$ and $Fe(CO)_5$, the metals have zero oxidation state.

Reason R : Low oxidation states are found when a complex has ligands capable of π-donor character in addition to the σ-bonding.

In the light of the above statements, choose the **most appropriate** answer from the option given below.

(1) **A** is correct but **R** is not correct

(2) **A** is not correct but **R** is correct

(3) Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**

(4) Both **A** and **R** are correct and **R** is the correct explanation of **A**.

Official Ans. by NTA (1)

Sol. Low oxidation state of metals can stabilized by synergic bonding so ligand has to be π-acceptor.

70. During the reaction of permanganate with thiosulphate, the change in oxidation of manganese occurs by value of 3. Identify which of the below medium will favour the reaction.

(1) aqueous acidic

(2) aqueous neutral

(3) both aqueous acidic and neutral

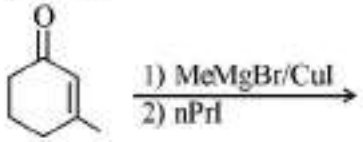
(4) both aqueous acidic and faintly alkaline.

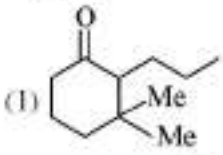
Official Ans. by NTA (2)

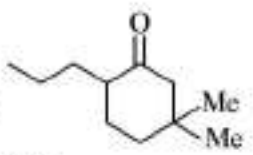
Sol. In neutral or weakly alkaline solution oxidation state of Mn changes by 3 unit

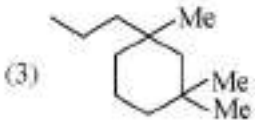
$$\overset{+7}{Mn}O_4^{-1} \rightarrow \overset{+4}{Mn}O_2$$

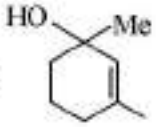
71. Find out the major product from the following reaction



(1) 

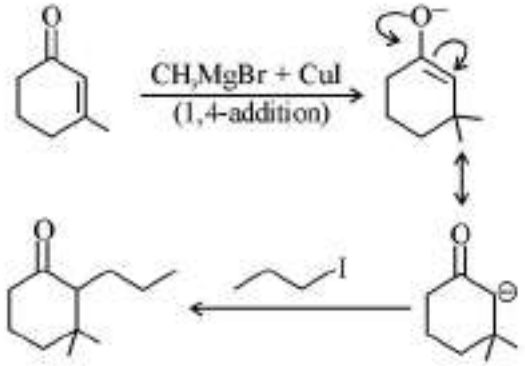
(2) 

(3) 

(4) 

Official Ans. by NTA (1)

Sol.



72. Formation of which complex, among the following, is not a confirmatory test of Pb^{2+} ions

(1) lead chromate

(2) lead iodide

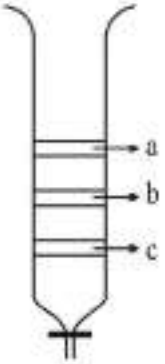
(3) lead nitrate

(4) lead sulphate

Official Ans. by NTA (3)

Sol. $\because$ $Pb(NO_3)_2$ is a soluble colourless compound so it cannot be used in confirmatory test of Pb^{+2} ion.

73. From the figure of column chromatography given below, identify incorrect statements.



A. Compound 'c' is more polar than 'a' and 'b'

B. Compound 'a' is least polar

C. Compound 'b' comes out of the column before 'c' and after 'a'

D. Compound 'a' spends more time in the column

Choose the correct answer from the options given below :-

(1) A, B and C only

(2) B, C and D only

(3) A, B and D only

(4) B and D only

Official Ans. by NTA (1)

74. Given below are two statements :-

Statement-I : Morphine is a narcotic analgesis. It helps in relieving pain without producing sleep.

Statement-II : Morphine and its derivatives are obtained from opium poppy.

In the light of the above statements, choose the **correct** answer from the options given below.

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are false

(3) Both Statement I and Statement II are true

(4) Statement I is false but Statement II is true

Official Ans. by NTA (4)

Sol. **Statement-I-** Morphine relieves in pain and produce sleep (incorrect)

Statement-II - Correct

75. The volume of 0.02 M aqueous HBr required to neutralize 10.0 mL of 0.01 M aqueous $Ba(OH)_2$ is (Assume complete neutralization)

(1) 2.5 mL

(2) 5.0 mL

(3) 10.0 mL

(4) 7.5 mL

Official Ans. by NTA (3)

Sol. $N_1v_1 = N_2v_2$

$\Rightarrow 0.02v_1 = 0.02 \times 10$

$\Rightarrow v_1 = 10ml$

76. The product, which is not obtained during the electrolysis of brine solution is

(1) NaOH

(2) Cl_2

(3) H_2

(4) HCl

Official Ans. by NTA (4)

Sol. Brine is aq. Solution of NaCl

$NaCl_{(aq)} \rightarrow Na^+ + Cl^-$

Cathode reaction

$2H_2O + 2e^- \xrightarrow{\text{reduction}} H_{2(g)} + 2OH^-$

Anode reaction

$2Cl^- \xrightarrow{\text{oxidation}} Cl_{2(g)} + 2e^-$

So HCl will not form during electrolysis.

77. The group of chemicals used as pesticide is

(1) Sodium chlorate, DDT, PAN

(2) Aldrin, Sodium chlorate, Sodium arsinite

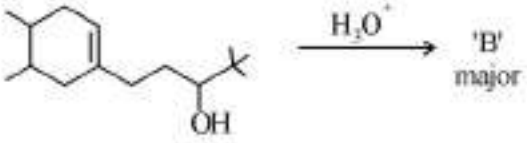
(3) DDT, Aldrin

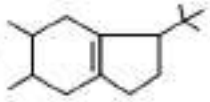
(4) Dieldrin, Sodium arsinite, Tetrachloroethene

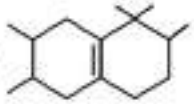
Official Ans. by NTA (3)

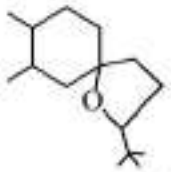
Sol. Pesticides → D.D.T and Aldrin

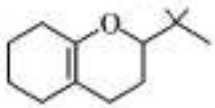
78. In the following reaction, 'B' is



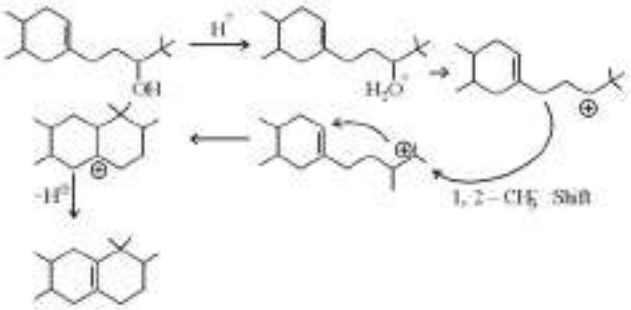
(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (2)

Sol. 

79. Which one of the following elements will remain as liquid inside pure boiling water ?

(1) Cs

(2) Ga

(3) Li

(4) Br

Official Ans. by NTA (2)

Sol. Li, Cs reacts vigorously with water.

Br_2 changes in vapour state in boiling water (BP = 58°C)

Ga reacts with water above 100°C (MP = 29°C, BP = 2400°C)

80. If the radius of the first orbit of hydrogen atom a_0, then de Broglie's wavelength of electron in 3rd orbit is

(1) $\frac{\pi a_0}{6}$

(2) $\frac{\pi a_0}{3}$

(3) $6\pi a_0$

(4) $3\pi a_0$

Official Ans. by NTA (3)

Sol. $(r_3)_H = \frac{a_0 n^2}{Z} = a_0 \times 3^2 = 9a_0$

$2\pi r = n\lambda$

$\Rightarrow 2\pi \times 9a_0 = 3\lambda$

$\Rightarrow \lambda = 6\pi a_0$

SECTION-B

81. In an ice crystal, each water molecule is hydrogen bonded to .....neighbouring molecules.

Official Ans. by NTA (4)

Sol. In ice each water molecule is hydrogen bonded with four other water molecules.

82. The equilibrium composition for the reaction $PCl_3 + Cl_2 \rightleftharpoons PCl_5$ at 298 K is given below.

$[PCl_3]_{eq} = 0.2 \text{ mol } L^{-1}$ $[Cl_2]_{eq} = 0.1 \text{ mol } L^{-1}$,

$[PCl_5]_{eq} = 0.40 \text{ mol } L^{-1}$

If 0.2 mol of Cl_2 is added at the same temperature, the equilibrium concentrations of PCl_5 is_____ $\times 10^{-2}$ mol L^{-1}.

Given : K_c for the reaction at 298 K is 20

Official Ans. by NTA (48)

Sol.

	PCl_3	+ Cl_2	$\rightleftharpoons$	PCl_5
	0.2M	(0.1+0.2)M		0.4M
Eq^m.	0.2–x	0.3–x		0.4+x

$$\frac{(0.4+x)}{(0.2-x)(0.3-x)} = 20$$

$\Rightarrow x = 0.086$

$[PCl_5]_{eq} = 0.486M = 48.6 \times 10^{-2} M$

83. Consider the following pairs of solution which will be isotonic at the same temperature. The number of pairs of solutions is/are.....

A. 1 M aq. NaCl and 2 M aq. Urea

B. 1 M aq. $CaCl_2$ and 1.5 M aq. KCl

C. 1.5 M aq. $AlCl_3$ and 2 M aq. Na_2SO_4

D. 2.5 M aq. KCl and 1 M aq. $Al_2(SO_4)_3$

Official Ans. by NTA (4)

Sol. $\pi = icRT$

A, B, C and D are isotonic pairs.

84. The standard reduction potential at 298 K for the following half cells are given below :-

$NO_3^- + 4H^+ + 3e^- \rightarrow NO(g) + 2H_2O$ $E^0 = 0.97V$

$V^{2+}(aq) + 2e^- \rightarrow V$ $E^0 = -1.19V$

$Fe^{3+}(aq) + 3e^- \rightarrow Fe$ $E^0 = -0.04V$

$Ag^+(aq) + e^- \rightarrow Ag(s)$ $E^0 = 0.80V$

$Au^{3+}(aq) + 3e^- \rightarrow Au(s)$ $E^0 = 1.40V$

The number of metal(s) which will be oxidized by NO_3^- in aqueous solution is _____.

Official Ans. by NTA (3)

Sol. Metal having lower SRP than 0.97V will be oxidised by NO_3^-.

85. The number of colloidal systems from the following, which will have 'liquid' as the dispersion medium, is_____

Gem stones, paints, smoke, cheese, milk, hair cream, insecticide sprays, froth, soap lather.

Official Ans. by NTA (5)

Sol. Paints, milk, hair cream, froth, soap lather.

86. The number of species having a square planar shape from the following is _____

XeF_4, SF_4, SiF_4, BF_4^-, BrF_4^-, $[Cu(NH_3)_4]^{2+}$, $[FeCl_4]^{2-}$, $[PtCl_4]^{2-}$

Official Ans. by NTA (4)

Sol. XeF_4, BrF_4^{-1}, $[Cu(NH_3)_4]^{+2}$, $[PtCl_4]^{-2}$ has square planar shape.

87. Consider the following date

Heat of combustion of $H_2(g) = -241.8$ kJ mol^{-1}

Heat of combustion of $C(s) = -393.5$ kJ mol^{-1}

Heat of combustion of $C_2H_5OH(l) = -1234.7$ kJ mol^{-1}.

The heat of formation of $C_2H_5OH(l)$ is (–) ________ kJ mol^{-1} (Nearest integer)

Official Ans. by NTA (278)

Sol. $2C_{(s)} + 3H_{2(g)} + \frac{1}{2}O_{2(g)} \rightarrow C_2H_5OH_{(l)}$

$(\Delta H_f)_{C_2H_5OH_{(l)}} = \Sigma(\Delta H_{comb})_{reactant} - \Sigma(\Delta H_{comb})_{product}$

$= 2 \times (-393.5) + 3(-241.8) - (-1234.7)$

$= -277.7 \text{ kJ / mol}$

88. Among the following, the number of compounds which will give positive iodoform reaction is________

(a) 1-Phenylbutan-2-one

(b) 2-Methylbutan-2-ol

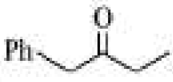
(c) 3-Methylbutan-2-ol

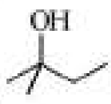
(d) 1-Phenylethanol

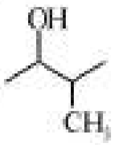
(e) 3,3-dimethylbutan-2-one

(f) 1-Phenylpropan-2-ol

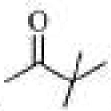
Official Ans. by NTA (4)

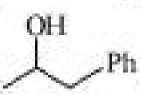
Sol. (a) 

(b) 

(c)  (Positive for iodoform)

(d) Ph–CH(OH)–CH$_3$ (Positive for iodoform)

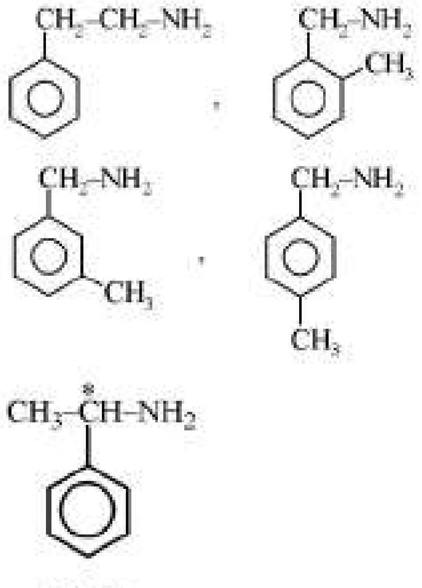
(e)  (Positive for iodoform)

(f)  (Positive for iodoform)

89. Number of isomeric aromatic amines with molecular formula $C_8H_{11}N$, which can be synthesized by Gabriel Phthalimide synthesis is________

Official Ans. by NTA (5)

Sol.



(d + l)

90. Number of crystal systems from the following where body centred unit cell can be found, is.....

Cubic, tetragonal, orthorhombic, hexagonal, rhombohedral, monoclinic, triclinic.

Official Ans. by NTA (3)

Sol. Cubic, tetragonal and orthorhombic have body centered unit cell.